The Dissertation Committee for Stephanie Ryan Nichols certifies that this is the approved version of the following dissertation:

Student-to-Student Discussions: The Role of the Instructor and Students in Discussions in an Inquiry-Oriented Transition to Proof Course

Committee:

Jennifer Christian Smith, Supervisor

Edward Odell

Walter Stroup

Lupita Carmona

Sherry Field

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by

Stephanie Ryan Nichols, B.A., M.S.

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Student-to-Student Discussions: The Roles of the Instructor and Students in Discussions in an Inquiry-Oriented Transition to Proof Course

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Stephanie Ryan Nichols, Ph.D.

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Supervisor: Jennifer Christian Smith

This study of student-to-student discussions focuses on a single inquiry-oriented transition to proof course. Mathematical proof is essential to a strong mathematics education but very often students complete their mathematics studies with limited abilities to construct and validate mathematical proofs (c.f. Harel & Sowder, 1998; Knuth, 2002; Almeida, 2000). The role of mathematical proof in education is to provide explanation and understanding. Both the research on mathematical discourse and the standards of the NCTM claim that participation in mathematical discourse provides opportunities for understanding. Although this link has been established, there is very little research on the role of students and the instructor during discussions on student-generated proofs at the undergraduate level – particularly in inquiry-oriented classes. This research analyzes the types of discussions that occurred in an inquiry-oriented undergraduate mathematics course in which proof was the main content. The discussions of interest involved at least two student participants and at least three separate utterances. These discussions fell along a continuum based on the level of student interaction. As a result of this research, the four main discussion types that were present in this course have been described in detail with a focus on the roles of the instructor and the students. The methodology for this research is qualitative in nature and is an exploratory case study. The data used for this research was video tapes of two to three class sessions per week of an Introduction to Number Theory course taught in the fall of 2005.

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1. Introduction

When thinking about mathematics it is impossible not to consider proof. Schoenfeld (1998) argues that "proof is not a thing separable from mathematics as it appears to be in our curricula; it is an essential component of doing, communicating, and recording mathematics" (p. 76). Most students complete their secondary mathematics work and even their undergraduate work with little or no understanding of what mathematical proof is or how a mathematical proof is produced (c.f. Harel & Sowder, 1998; Knuth, 2002; Almeida, 2000). This is in spite of the recommendations of the National Council of Teachers of Mathematics (NCTM) which in 2000 released standards that included proof and reasoning as one of the five core standards. Furthermore, this lack of experience in proof is in contradiction to what mathematicians actually do. For students to truly learn mathematics, mathematical argumentation and proof must be essential parts of their school mathematics experience. Thus studying the teaching and learning of proof is a key component in the improvement of the teaching of mathematics overall.

Mathematical proof is a difficult concept to define clearly. In fact Lakotos (1976) argues in his *Proof and Refutations* that no proof is truly final and thus identifying what a proof is can ultimately depend on the community for which the proof is intended. One common definition is that a mathematical proof is a logical argument that establishes the truth of a conjecture. Another is that a mathematical proof is an explanation of the truth of a statement. While it is difficult to find consensus within the mathematical community as to what the definition of proof is there is some consensus in the literature as to the roles of proof. Knuth (2002) gives these roles as "1) to verify that a statement is true, 2) to explain why a statement is true, 3) to communicate mathematical knowledge, 4) to discover or create new mathematics, or 5) to systematize statements into an axiomatic system" (p. 381). The roles of proof “to explain why a statement is true” and “to communicate mathematical knowledge”, in particular, support studying the teaching and learning of proof in a classroom in which the aspect of explanation and communication are valued.

In addition to the roles of proof, the ways in which mathematicians determine the validity of a proof also provide insight into how proof might be taught. Hanna (1991) describes what factors mathematicians find important in deciding whether to accept a proof as valid. She describes this as a social process where the significance of the theorem and the understanding of the proof are more important than the rigor of the proof. This point of view is especially relevant to my research because in the classroom I have studied the acceptance of a proof is entirely a social process. Hanna (1991) argues that most mathematicians will accept a theorem or proof as valid if they understand the theorem, it is consistent with what are already accepted mathematical results, or there is a convincing mathematical argument for it. There are some other criteria mentioned in the literature, but the ones given link the idea of mathematical proof to the social aspect of the proof process as well as the acceptance of a proof as valid. Thus both the roles of proof and the manner in which a proof is determined to be valid in the mathematical community support studying the learning of proof in a classroom that embodies the ideas of proof as a social process that involves communication and explanation.

My interest in Dr. Matthew Stone[1] and his form of inquiry-oriented teaching of proof began in the fall of 2002. I was enrolled as a student in his Introduction to Number Theory course taught using a version of the modified Moore method[2]. This was the first time I had been enrolled in a course where proof and, more importantly, student proof was the focus. My interest has continued to grow through my appointment as a teaching assistant to other experienced modified Moore method instructors as well as those who are teaching Number Theory with this type of instruction for the first time.

In the fall of 2003, in addition to serving as a teaching assistant to Dr. Stone, I was part of a group of researchers who studied his Number Theory course. During this semester we collected video and interview data and in the following semesters began to analyze these data. Within this research project I conducted a preliminary analysis of the instructor's role in terms of asserting himself as leader and how these actions related to instances of encouraging participation, offering strategies for proof, and facilitating discussions of proofs. I presented this analysis in a poster session at PME-NA in Toronto, Ontario (Nichols & Smith, 2004) and gave a talk at the RUME conference in Phoenix, Arizona (Nichols & Smith, 2005). It was this analysis that has led to my dissertation research.

With this research I hope to expand both my and the research community's' understanding of the roles that both the instructor and students play throughout the

1 Dr. Matthew Stone is a pseudonym and will be used throughout this dissertation.
2 The modified Moore method is a type of inquiry-oriented teaching in which students are given no text but rather a set of theorems and questions that they are required to prove outside of class. The students then present their solutions during class and the other students critique their work through discussion. The instructor's involvement varies based on the instructor as well as the circumstances of the class. A more thorough description of Dr. Stone's class will be given in the methodology chapter.

semester in a class in which discussion, the formation of a collaborative community, and understanding are norms and are considered integral parts of the proof construction process. This research focuses in particular on the types of discussions that occur in an inquiry course and the role of the instructor and students in these.

My research question is: What are the roles of the instructor and students throughout a semester in the discussions that occur in an inquiry-oriented course where the roles of proof to communicate and explain mathematics are valued as integral parts of the proof construction and validation processes?

I have identified several main pieces of research that are relevant to my research question. The first of these is the body of research on undergraduate understanding of proof. A large part of this research argues that undergraduates and teachers lack a deep understanding of proof and have an inability to create mathematical proofs (Harel & Sowder, 1998; Knuth, 2002; Almeida, 2000). There is a growing area of research in undergraduate mathematics education that studies how students think about proof and how metonymy, diagrams, and instrumental and relational understanding both aid and hinder students in their production of mathematical proofs (Gibson, 1998, Zandieh & Knapp, 2006).

The second area of research focuses on classrooms that are inquiry-oriented in nature, where the ideas of collaboration, discussion, and understanding are valued. There are very few pieces of research that study these types of classrooms at the undergraduate level. Those that do, which include work by Rasmussen and Yackel, do not have proof as the focus or are anecdotal in nature (c.f. Dean, 1996, Levin & Shanfelder, 2000, Shabel, 2005). The majority of these studies take place at the elementary and secondary level (Woods, 1999; the work on CGI, Carpenter, Fennema, Franke, Levi, and Empson, 1999; Goos, 2000). Thus those classes that have been studied at the undergraduate level have not been thoroughly explored when it comes to teaching and learning the proof process.

Connected to this is the research that looks at undergraduate classrooms where proof is a part of the content and in doing so study student understanding of proof. Many of these classrooms valued discourse and argumentation. The work of Erna Yackel (2002), who studied the role of collective argumentation in an undergraduate differential equations course taught by Chris Rasmussen, reflects the type of research done at this level. In addition, further research has been done on Differential Equations courses that explore the use of gesturing as well as a general analysis of the collective learning process that occurred through argumentation (Rasmussen, Stephan, & Allen, 2004, Stephan & Rasmussen, 2002). Another example of research conducted on undergraduate mathematics courses taught in an inquiry-oriented manner is the research of Rasmussen, Zandieh, King, & Teppo (2005). Rasmussen, et al. (2005) studied the development of advanced mathematical thinking in undergraduate students enrolled in Differential Equations classes as well as students in Geometry classes.

The final piece of the literature focuses on the study of discussion or discourse in mathematics classes at all levels. Some of this research focuses on the statements that students and teachers make and what this implies about student understanding (Cobb et al., 1997, Knuth & Peressini, 2001, Wood et al., 1993) while others relate the content of the discourse to student beliefs about mathematics (Ju & Kwon, 2007). An overall theme to the research on discourse is that participating in meaningful discourse is linked to student understanding (Sfard & Kieran, 2001; Wood et al., 1993; Yackel & Cobb, 1996). I have chosen to focus on the structure of discussions involving two or more students as well as the roles of both the students and instructor in these discussions in an inquiry-oriented undergraduate mathematics class.

While the discussions that elementary and secondary students have about mathematical justifications that they have created may be similar to those that occur at the undergraduate level, there is very little research on the types of discussions that undergraduate students have about their own mathematical justifications or proofs. At the undergraduate level these solutions are mathematical proofs. Thus my research builds on the research connecting student understanding with discourse but fill a gap in the research in terms of the structure of these different types of discussions as a part of the proof process.

1.1 Inquiry-Oriented Learning and the modified Moore method

Dr. Stone has generally referred to his Number Theory course as an inquiry-oriented course. However, the course is also referred to as a modified Moore method course due to Dr. Stone's own experiences in Moore method courses. There is a long history behind the modified Moore method and often the theoretical or pedagogical perspective of an instructor of a modified Moore method course is in contrast to that of R.L. Moore, whose particular teaching style the modified Moore method is based on.

1.1.1 The R.L. Moore Method

The basic idea behind the Moore Method is that “bright students can develop mathematics by working individually within a competitive system” (Dancis & Davidson, 1970). The heart of R.L. Moore’s method was a sequence of problems and theorems that had been carefully selected in order to enable students to prove and present this material to their class (Mahavier, 1997). In Moore’s method the steps that were necessary to go through the sequence of theorems and exercises were close to the maximum size for the class; since his students were given several days to answer the questions (Smith, 1996). The aim of this method was to develop both knowledge and research ability (Wilder, 1976).

R.L. Moore developed the Moore Method for teaching mathematics from 1920 to 1969 at the University of Texas. R.L. Moore had the luxury of being able to select the students that could take his classes. If a student had studied the subject before, then Moore would exclude the student from his class. Moore made his selections of students through personal interviews or past performance in courses taught by his methods (Dancis & Davidson, 1970). Moore wanted to have a class that was as “homogeneously ignorant” as possible (Jones, 1977, p. 273). In Moore’s classes competition was a driving force and he wanted the competition to be as fair as possible.

Moore’s method was a complete system in that it worked for all the classes he taught from trigonometry and calculus all the way through advising PhD dissertations (Dancis & Davidson, 1970). Moore would give the axioms that would be used in his class and then give examples that illustrated their meaning. He then would give some definitions and theorems without explanation. The students were then responsible for finding proofs of their own to these theorems and also to construct examples to show that the hypotheses of the theorems could not be weakened, omitted, or partially omitted (Jones, 1977). When the students returned the next day Moore would call on some students to prove the theorems. Once Moore knew the abilities of his students he would always ask the unsuccessful students first to present a proof. When a student was presenting the other students were responsible for making sure that the proof that was presented was correct and convincing (Jones, 1977). This allowed for friendly competition among the students to impress Moore and to solve more of the difficult problems than their classmates did (Dancis & Davidson, 1970).

One of Moore's former students, Martin Ettlinger, "described the atmosphere in Moore's classes as extraordinary" (Renz, 1999, p. 2). Ettlinger said that every student's ideas were listened to both carefully and critically and no discourtesy was tolerated. However, every idea was tested before being accepted (Renz, 1999). Moore's students learned the importance of checking their work carefully and to present their work clearly. Moore used student presentations as a tool to build the students' abilities to "monitor and improve their own work and to give them the confidence to stand up and present their ideas" (Renz, 1999, p. 3).

The success of R.L. Moore's Method speaks for itself. During his time at the University of Texas at Austin, Moore supervised 50 PhD students. Of these fifty are "two former presidents of the American mathematical Society, four former presidents of the Mathematical Association of America, three members of the National Academy of Sciences" (Wilder, 1976, p. 417).

1.1.2 The modified Moore method

There are as many versions of the modified Moore method (MMM) as there are instructors and courses that use the modified Moore method. In general, the approach to teaching and learning mathematics represented by the modified Moore method emphasizes problem solving and sense-making over transmission of information, and so shifts the focus in the classroom from the actions of the teacher to the actions of the students. In addition, the level and pace of the course is adjusted to the students based on the teacher's perception of students' understandings and conceptions of mathematical ideas (W. S. Mahavier, 2000; W. T. Mahavier, 1997). The modified Moore method is based on the idea that students construct meaningful mathematical knowledge when they engage in solving carefully selected problems. By having students solve a variety of problems and explain their thinking classroom activity is focused on the development of students' mathematical understanding and creative thinking (W.T. Mahavier, 1997). Finally, the roles of students and teachers are considered important and interdependent in the MMM. As an active learner, the student is expected to participate in the learning process and to be responsible for his or her learning. As a facilitator, the teacher is expected to guide students' learning, pose questions and assist group work and classroom discussion (W.T. Mahavier, 1997).

While all versions of the modified Moore method share these common views they vary in how much they differ from the original Moore method. In my personal experience with several instructors who use the modified Moore method, I have found many different interpretations. The method that Dr. Stone uses does not appear to rely on competition, rather a sense of a community approaching a common goal. Due to the many differences between methods and Dr. Stone's own propensity to do so, I will refer to this course as an inquiry-oriented course in order to avoid any misconceptions as to what the modified Moore method looked like in Dr. Stone's Number Theory course.

1.2 Prior Work

During the summer of 2004 I chose to focus my attention on the role of the instructor from the 2003 data. This analysis focused on three main roles and behaviors of the instructor that were found during the coding process. These were when the instructor assumed the role of leader, used positive feedback, and developed rapport[3]. We found that the instructor purposefully relinquished his role as leader during class sessions, both to the whole group and to presenters via: his position in the classroom, his voice (tone, volume, and silence), his physical gestures, and the types of comments or questions he asked. We found that the instructor rarely commented that a presented proof was correct or incorrect. He made positive comments that appeared to contribute to a sense of accomplishment in the class as well as pride in their work, emphasizing the community's pursuit of a common goal. The last behavior I focused on was developing rapport. The instructor established a friendly and open environment from the very first class meeting. The result was an environment in which the students felt comfortable expressing doubt and lack of understanding as well as supporting their classmates. We found that while

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3 The coding matrix that was used during the analysis of the 2003 data can be found in the appendix. The three codes that I focused my analysis on are highlighted.

each member played a vital role the instructor was vital in establishing the roles of the other members of the community. In addition, through decisions of when to assert his presence and when to "hide in the back", the instructor began to put the authority to determine whether proofs were acceptable or not into the hands of the community. The atmosphere that the instructor developed allowed students to be more vocal and active than what might be seen in other traditional undergraduate mathematics classes. I presented this research in a poster session at PME-NA in Toronto, Ontario in the fall of 2004.

I chose to continue this analysis of the 2003 video data by focusing on one of these three behaviors. I was interested in investigating the instances that the instructor, Dr. Stone, both assumed and relinquished his role as leader and why he might choose to do so.

In the beginning of the semester most of the instances in which the instructor asserted himself as leader occurred during whole class discussions or when he was trying to get volunteers to present. During this part of the semester the problems were easier which meant that issues related to the mathematical community norms or rules rarely came up, which in turn meant the instructor did not assert himself as leader for these types of issues.

Towards the middle of the semester there are far more instances of the instructor assuming the role of leader and much more variety in the situations during which this occurred. There were more interactions during which the instructor asserted himself as leader in the form of the teacher addressing the class, the teacher lecturing, as well as whole group discussions, which in many cases were discussions involving strategies for proof. Besides these types of interactions there were more times when weaker students were presenting or when the proof presented was incorrect. This may have effected the decision of the instructor to assume the role of leader more often. Also during this segment of the semester induction was introduced and formalized which resulted in more teacher leadership.

At the end of the semester the number of instances of the instructor asserting himself as leader was fewer than in the middle, as was also the case in the beginning of the semester. The times when the instructor asserted himself as leader occurred along with motivating participation, offering strategies for proof, and facilitating discussions almost equally, unlike the first part of the semester. Overall most occurrences of the instructor asserting himself as leader involved difficult proofs. However there are just as many difficult proofs in this part of the semester as the previous, but the teacher asserted himself far fewer times.

I presented this analysis at the RUME conference in Phoenix, Arizona in February of 2005. This analysis led to my interest in further studying the role of the instructor and coming to a better understanding of the roles he plays in developing this community and how these change throughout the semester.

2. Theoretical Perspective

In this section I will describe my theoretical perspective and how it supports my research. Although my research is loosely based on grounded theory, I have included a discussion of sociocultural theory both in how it informed my theoretical perspective prior to my research and the piece that I added after my analysis led to a focus on discourse. The portion of this theory related to discourse was unknown to me at the time of my analysis and is only included now because of it's relevance to my research.

2.1 Sociocultural Learning Theory – Participation in a Community

The theoretical perspective that I took, both in the formation of my research question and initial analysis, is that of sociocultural learning theory. This theory traces its roots to the work of Vygotsky and others in the early 20th century who claimed that human thinking is inherently social (Forman, 1996). Vygotsky believed that higher mental functioning in the individual derives from social life (Vygotsky, 1987, Wertsch, 1991). He also believed that social processes are an essential part of learning even if this learning does not take place in a social context. Vygotsky believed that independent activity occurs after the social processes of higher mental functioning have been internalized. Thus even independent activity is social in nature (Tudge, 1989).

Vygotsky also argued that learning took place within an individual's zone of proximal development and that this depended on social interaction within a framework based on the shared culture of the activity (Nicolopoulou, 1993, Vygotsky, 1987). He argued that analysis on learning needed to integrate the thoughts and actions of individuals with their culturally specific activities (Forman, 1996, Vygotsky, 1987).

In addition to the work of Vygotsky, the work of Lave has contributed to sociocultural theory. Lave described learning as a process by which someone that is unknowledgeable is integrated into a community of practice. She called this process of integration legitimate peripheral participation (Lave, 1988). She, like Vygotsky, argued that learning was increasing participation in communities of practice (Goos, 2004). Therefore, what matters as far as learning is one's access to meaningful participation in a community of practice. She felt that the classroom teacher plays a very important role in a community of practice within the school. The teacher is the knowledgeable member of the community who communicates the norms, values, and discourse practices of the community to the students who are unfamiliar with them. The teacher is also essential in designing meaningful settings in which students can be active participants. Lave says it is possible to learn math by doing what mathematicians do, by engaging in structure-finding activities and math argumentation typical of good mathematical practice. She claims the processes of learning and understanding are socially and culturally constituted (Lave, 1988, Forman, 1996).

2.2 Sociocultural Learning Theory – Language and Discourse

A large part of sociocultural learning theory is the tool that is used by learners to participate in the social processes and therefore be able to transfer this knowledge to the individual level. Of course, even on the individual level this knowledge is social in nature. The tool that is used is language and discourse (Nicolopoulou, 1993).

Yakubinsky makes a distinction between two different types of discourse: dialogue and monologue. He describes dialogue as utterances succeeded by other utterances; as “mutual interruption” (p. 250, Yakubinsky & Eskin, 1997). He claims that dialogue is the natural form of speech due to the more simple composition of dialogue. He states that when people are involved in dialogue there are not as many words necessary due to the implied meanings mutually understood by those involved (Yakubinsky & Eskin, 1997). Stroup et al. (2007) explain this further, stating that there is less effort and fewer words needed because of a common understanding shared by the participants in dialogue. Yakubinsky calls this shared understanding the *apperceptual mass* and claims that the greater the shared mass the less need there is for language to convey meaning. He states that in dialogue the shared apperceptual mass is greater than in monologue.

Yakubinsky argues that without dialogue there is no interaction in the speech. With monologue everything is thoughtfully spelled out and therefore is more artificial in nature. Yakubinsky contends that with monologue the listeners are passive only making conciliatory responses. This is a clear connection between monologue and the idea of an authority (Yakubinsky & Eskin, 1997).

Wertsch (1985) claims that Vygotsky was influenced by Yakubinsky’s views on dialogic and monologic speech. Vygotsky used similar descriptions when describing dialogic speech. Wertsch (1985) quotes Vygotsky describing dialogic speech as “simplified syntax, syntactic condensation, the expression of a thought in condensed form, significantly fewer words” (p.88). Vygotsky saw speech to have individual and social functions - both communicative and intellectual functions - and that these are inherently interconnected (Wertsch, 1985).

Ultimately, these types of speech present themselves in classrooms. Stroup et al. (2007) argue that when students are introduced to novel science and mathematics concepts there is a relative absence of the apperceptual mass. In addition they note that most science and mathematics classroom practice is centered on monologic discourse. They argue that this means that many students, especially when novel content is introduced, are excluded from participating in a meaningful way. They argue for more natural and dialogic speech in classrooms in order to support shared understandings (Stroup et al., 2007).

2.3 Conclusions

This perspective on learning supports my research in several ways. The first is that sociocultural learning theory contends that learning mathematics is a social activity in which communication is a major tool. The contrast of monologic speech and dialogic speech support my analysis of the types of discussions that occur along a continuum between the two. In addition, the work of Stroup et al. (2007) argues that classrooms should be dominated by participation in more dialogic speech. This research hopes to capture what type of discourse occurs in an inquiry-oriented classroom in order to explore the notions of monologic and dialogic speech where the subject is undergraduate students' mathematical artifacts – proof.

3. Review of Relevant Literature

In this section I will review the research regarding the philosophy of proof focusing on the lack of a concise definition, the multiple roles of proof, and the social nature of validating proof in the mathematical community and how these tie into how proof should be taught and learned in the mathematics classroom. I will then move on to discuss the research on student understanding of proof focusing on the literature that classifies student proof schemes, the research that studies student proof perceptions and construction processes from a deficit view, and those that seek to understand what challenges these students face during this process and alternatively what makes them more successful. A small piece of this research studies the proof perceptions and construction processes of students in inquiry-oriented classes and links to the body of research on inquiry-oriented classes. This research is dominated by studies at the elementary level or those for which proof is not the central content being taught. My final piece of relevant literature is a body of research which studies mathematical discourse. This research links participation in mathematical discourse with understanding and mainly focuses on analyzing discourse in order to describe the level of understanding of the students involved. Finally I will tie all of these together noting that in each area of research there is a lack of research on student discussion in an inquiry-oriented course where students are learning to construct and validate mathematical proof.

3.1 Philosophy of Proof

For the purposes of this literature review I will focus on three pieces of the literature on the philosophy of proof. These are the definition of proof, the role of proof, and validating proof. It is clear from the literature that there is not an accepted, concise definition of proof. I have found that for the purpose of my research looking at the role of proof and validation of proof offer more useful information in regards to the teaching of proof.

The research on mathematical proof and the role it plays in both the teaching and doing of mathematics is varied. However most tend to agree that the role of proof is multi-faceted and that the role of proof in the mathematical community is in contrast to the role of proof in education. In addition there is a body of literature that discusses what it means for a proof to be considered valid. There is support for the view that there is not one concise answer, but rather a range of criteria that are deemed important when determining the validity of a mathematical proof by the mathematical community.

3.1.1 Definition

Defining mathematical proof is a difficult task. What constitutes a proof varies depending on the community the proof is intended for. In addition, mathematical proof can be seen in two ways, as the actual process one goes through in constructing a proof and the final product. Although there is a lack of an accepted and operational definition of proof Hersh (1993) gives a definition of proof which involves three different parts.

These are:

(1) to test, try out, determine the true state of affairs,
(2) an argument that convinces qualified judges, and
(3) a sequence of transformations of formal sentences, carried out according to the rules of predicate calculus (p. 391).

Hersh (1993) claims that the second meaning is the one that is common in the mathematical community whereas the third is the one that is common to mathematical logic and the philosophy of mathematics. The second of these definitions relates to the social nature of the validation process of a mathematical proof which is relevant to my research question. In addition we will see this definition relates closely to what is described as one of the roles of proof.

3.1.2 Role of Proof

Generally the role of proof gives more insight into what a proof truly is rather than the definition because it is difficult to explain what constitutes a proof more specifically than the definition given. Five roles of mathematical proof are described in the literature. Knuth (2002) gives these roles as "1) to verify that a statement is true, 2) to explain why a statement is true, 3) to communicate mathematical knowledge, 4) to discover or create new mathematics, or 5) to systematize statements into an axiomatic system" (p. 381). Hanna and Jahnke (1996), while agreeing with these five categories, argue that the communication of mathematical understanding is the most significant role of proof in terms of mathematics education. Hersh (1993) argues that in the mathematical community the purpose of proof is to convince. The role of proof to communicate mathematics supports studying the discussions about mathematical proof that occur over the course of a semester in a class that’s main purpose is to teach proof. In addition the role of proof to explain supports studying how students talk about their proofs and especially how they explain their proofs to each other through their discussions.

3.1.3 Validating Proof

These five roles also have a bearing on the manner in which mathematicians, in general, determine if a proof is valid. These further reinforce how mathematical proof should be treated in education. Hanna & Jahnke (1996) give several criteria that mathematicians use when accepting a proof as valid. These include that (1) they understand the theorem and (2) that there is a convincing mathematical argument for it. They also contend that producing a rigorous proof is not always considered more important than understanding the proof and the significance of the conjecture. Similarly, Hersh (1993) notes that in the mathematical community determining the validity of a proof is dependent on whether or not it convinces a community of qualified judges. He also argues that formal proof is many times an impossibility and that they are usually used to verify the truth of a part of a more informal proof. In general, mathematicians are more concerned with the value of the results rather than the rigor of the argument.

Thus, proof can be interpreted as a social activity in which the input of the community is essential. It is also clear from the research that the validity of a proof relies on that proof convincing a relevant community. This implies that when studying the learning and doing of proof we should focus on whether or not these proofs convince and promote understanding and should study classrooms in which these views of the role of proof are accepted and practiced. In addition the discourse used to convince, explain, and validate is a large part of the social activity and is an essential component in the research on teaching and learning proof.

3.1.4 Role of Proof in Education

I have presented research describing the role of proof and how mathematicians determine validity, but I have yet to support the relevance of proof in mathematics education and thus for research on proof within mathematics education. As I stated in my introduction, mathematical proof is the hallmark of mathematics. Many mathematicians and researchers have argued that proof is an essential part of mathematics (c.f. Schoenfeld, 1998). In terms of mathematics education, it is clear from the NCTM Standards that proof is a crucial part of a strong mathematical understanding. In the 2000 Standards (Carpenter & Gorg, 2000) there is separate strand for *Proof and Reasoning*. This particular strand recommends that every student, from K-12, should be able to:

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

(Carpenter & Gorg, 2000, p.56)

Although these recommendations are for grades K-12, this standard emphasizes the role that proof plays and should play in mathematics education. Research has shown that more often than not these standards are not met at the secondary level (c.f. Healy & Hoyles, 2000), which makes the issue of proof at the undergraduate level even more important. I will further discuss the issues of student understanding of proof in the next section.

3.2 Student understanding of proof

There is a large body of research looking at student understanding of proof. The literature tends to fall into three main categories: 1) those that look to categorize student proof schemes, 2) those that show student inability to construct proofs and 3) those that seek to try and understand why students have difficulty with mathematical proof or what tools students who are successful seem to have at their disposal.

3.2.1 Classification of Proof Schemes and Student Lack of Understanding

The first two of these categories can be seen as a deficit view and has been somewhat widespread in mathematics education research. Generally, this research shows that most students (mainly undergraduate) are producing proofs that are based on specific examples or other types of invalid proofs. While some of this literature simply seeks to determine different proof schemes of secondary and undergraduate mathematics students (c.f. Harel & Sowder, 1998, Knuth & Elliott, 1998) others use this classification to conclude that these students have difficulties in constructing proofs (c.f. Recio & Godino, 2001, Segal, 2000, Hart, 1994). Harel & Sowder (1998) developed a classification of proof schemes that center around three main types: 1) external conviction, 2) empirical, and 3) analytical. They offer examples of each proof scheme and argue that it is not a hierarchical model and that students do not always work within a single proof scheme. Knuth & Elliott (1998) offer classifications of proof schemes for high school students as an aid for those who teach mathematical proof. They structure their classifications around Balacheff's framework.

While these researchers seem to be focused on classifying students' proof schemes others have studied students' perceptions of proof as well as their proof construction processes. Many of these conclude that students have difficulty with constructing proofs, most especially with deductive proofs. Recio & Godino (2001) studied undergraduate students' proof schemes within the context of mathematical content that the students were familiar with. They found that less than half of these students gave substantially correct proofs. They conjecture that this is related to the different institutional meanings of proof.

Segal (2000) was interested in studying undergraduate's perceptions of proof. He found a difference between what they found personally convincing and what they found to be valid. In addition he concluded that they were unable to distinguish between a valid and invalid proof. Segal (2000) argued that the students in his study considered most empirical arguments as convincing but not valid. In addition, like Recio & Godino (2001), he found that these students were unable to distinguish between correct and incorrect deductive arguments.

Coe and Ruthven (1994) studied students in their first year in a sixth-form college in the United Kingdom. They explored students' conceptions of proof generally, their views of the functions of proof, and their views on insight and understanding. Similarly to the other research, the authors concluded that students' proof strategies were mainly empirical in nature with very few incidences of deduction.

The work of Almeida (2000) further reinforces this deficit view but offers a suggestion for improvement. Almeida (2000) studied undergraduate's perceptions of proof and their actual proof practices. He found that they relied on informal and visual methods of proving. He conjectured that strong concept images were a key to improving proof productions.

There is also similar research on secondary students' conceptions of proof. Healy and Hoyles (2000) studied 14 and 15-year old students in algebra about their perceptions of proof as well as their abilities to construct proofs. They found that these students tended to have dual views of proof as those that would be acceptable to turn in for homework, i.e. satisfactory to some outside authority, and those that they found convincing. They also found that students tended to use empirical arguments when they constructed proofs.

This research generally shows that students are unable to construct mathematical proofs even if the proofs are over content they are familiar with. On the other hand there is a growing body of research that is interested in what causes these students to have difficulty with constructing proofs or point out strategies that aid students in being more successful in the proof construction process.

3.2.2 Understanding Student Difficulty and Success

Hart (1994) compared stronger students, in terms of proof construction and perceptions, with weaker students to try and understand why these weaker students had difficulty with proof. He found that the higher achieving students had the ability to implement and modify general problem-solving strategies as domain-specific strategies and had better control as to which strategies to use. In addition, Hart (1994) found that the weaker students demonstrated operation confusion which he related to an unstable conceptual schema or concept images. This research ties in with the research of Weber (2001) who compared the proof strategies of advanced mathematicians (professors and doctoral students) with those of undergraduate students.

Weber (2001) investigated student content knowledge and a potential disconnect between this knowledge and the construction of proofs. He found that undergraduates often know the facts required for the proof but are unable to prove the conjecture. He compared this type of knowledge, "strategic knowledge", in undergraduate students and doctoral students. He found that the doctoral students knew how to choose which facts and theorems to apply in a given proof, whereas the undergraduate students did not.

Gibson (1998) studied undergraduate students' use of diagrams in the proof construction process. He found that students generally used diagrams after they became stuck in order to 1) judge the truth of a statement, 2) understand information, 3) discover ideas, and 4) write out ideas. He found that although these students did not always successfully complete the proof (there was limited time for the interview) the students who used diagrams were more successful at completing subtasks of the proof construction process.

The work of Smith (2006) seeks to being to understand how instruction might influence student proof perceptions and proof construction strategies. She compared undergraduate students in traditional, lecture-based courses with undergraduate students in an inquiry-oriented course. She found that there were marked differences between these students and that the inquiry-oriented students tended to have more mature understandings of mathematical proof and sought to make sense of the mathematics. She conjectured that inquiry-oriented courses may encourage more meaningful approaches to proof.

Weber & Alcock (2004) analyzed data from two previous studies and categorized proof productions as either syntactic, one "draws inferences by manipulating symbolic formulae in a logically permissible way" (p. 209), or semantic, one "uses instantiations of mathematical concepts to guide the formal inferences that he or she draws" (p. 209). They used this data to describe the types of knowledge needed for both types of proof and found that these are quite different. They did not argue that semantic proof productions are superior to syntactic but did point out drawbacks of only being capable of syntactic proof productions. They also argue that syntactic proof productions are generally convincing without being explanatory whereas semantic proof productions are both.

Although exploring what students can do in terms of mathematical proof is a valuable area for research, it does not necessarily give insight into the teaching of proof. This research has focused on the students and not on the instructors who are creating classrooms where students are developing more mature understandings of mathematical proof. The research by Smith (2006) is the only research that has been conducted on an inquiry-oriented course where proof is the focus and it only studied student proof conception and ability. My research would study both the instructor and the students and the roles they played in discussions in an inquiry-oriented class – not whether or not the students can prove.

3.3 Research on Inquiry-Oriented Classrooms

There is body of research that is set in classrooms in which mathematical argumentation and/or mathematical proof are key components. Most of these classrooms would be considered inquiry-oriented in that student ideas and solutions are the focus of the classroom activity. Many of these focus on the class as a collaborative community and the instructor's role in these classrooms. The majority of this research has been done at the elementary level and looks at how teachers develop mathematical norms in their classrooms. This research has found that the role of the teacher is crucial in developing these norms and although each focused on different norms all found that discussion and understanding were norms that these teachers worked hard to establish.

Cognitively Guided Instruction (c.f. Carpenter, Fennema, Franke, Levi, & Empson, 1999) is an example of an inquiry-oriented classroom community at the elementary level. In this kind of classroom the role of the instructor is to facilitate discussions, pose problems, and guide the community in the development of mathematical understanding.

Yackel and Cobb (1996) spent time in a second grade classroom in which sociomathematical norms were being developed. They studied the processes that took place to establish and develop these norms as well as how these influenced mathematical argumentation in this classroom. They found that the teacher's role was central to establishing these norms because the teacher represented the mathematical community. Therefore the teacher's mathematical knowledge and understanding were crucial to the development of these norms and to student understanding.

Woods (1999) looked at the role of a teacher in a second grade classroom. Woods examined the ways in which the teacher helped to develop a community of inquiry where discussion and disagreement were valued. Woods found that in this classroom the teacher put a lot of effort into establishing the norms for discussion, participation, listening, and disagreement in this classroom community and her level of participation declined throughout the course of the year.

Similarly, McClain and Cobb (2001) studied the role of a first grade teacher in developing sociomathematical norms. In particular they focused on the role of discussion and the teacher's way of symbolizing student reasoning. They found that through the teacher's active guidance the students developed a sense of intellectual independence as well as a stronger mathematical disposition.

Since this research took place at the elementary level, it is clear that the classrooms being studied did not teach mathematical proof, per se. However, the research done on these classrooms is relevant to my research because these classrooms valued justification of mathematical ideas, which is a precursor to more formal mathematical proof and a focus of my research. In some of this research the norm of discussion was studied, however the discussions were not analyzed deeply, rather how this norm developed was the focus. In addition all of these researchers found that the teacher's role was key in developing these classroom norms.

While the majority of this research has been at the elementary level, the work of Marilyn Goos (2004) offers insight in to the instructor's role in a classroom community in eleventh and twelfth grade math classes. Goos found that the secondary teacher worked to establish the norm for discussion and emphasized the need for students to explain and justify their thinking, to be responsible for their own understanding of the mathematics involved, and to be active participants in the classroom activities. This also supports the notion that in these types of inquiry-oriented classes, the role of the instructor is central as well as the role of discussion.

There was very little research at the undergraduate level on inquiry-oriented classrooms until recently. However, in the research done on these types of classrooms proof is generally not the focus nor is the role of the instructor or students (c.f. Yackel, 2002). Moreover, much of the research on teaching in these types of classes is anecdotal in nature simply describing a particular method of teaching or a personal description of their experience teaching this type of class (c.f. Dean, 1996, Levin & Shanfelder, 2000, Shabel, 2005).

Yackel (2002), in addition to studying an elementary classroom, spent time in an undergraduate differential equations class in which argumentation was valued and proof and justification were an integral part of class discussions. Yackel (2002) was interested in the teacher's role in collective argumentation. She found that the instructor was responsible for initiating and fostering classroom mathematical norms that supported argumentation. In addition, Yackel (2002) argued that it is crucial for teachers in this type of classroom to have a deep, conceptual understanding themselves as well as a thorough understanding of the students' conceptual understanding. Although Yackel (2002) studied discussion in an inquiry-oriented class, she did not focus on the role of the students in these discussions, beyond what that implied in terms of the instructor’s role throughout the semester in developing the norm of argumentation.

Rasmussen et al (2004) studied undergraduate, differential equations classes to better understand the use of gesturing by the students. Although this class was an inquiry-oriented class, neither proof nor the instructor's role in fostering an understanding of proof was the focus. Prior to this work, Rasmussen et al. (2002) conducted a more general analysis of the classroom mathematical practices in this class and focused on the collective learning of the students especially through argumentation.

A final example comes from the work of Kwon, Rasmussen, & Allen (2005). Kwon et al. (2005) conducted a comparison of Differential Equations students from inquiry-oriented courses and traditional lecture-based courses. They found that the inquiry-oriented students, unlike the traditional students, retained their conceptual knowledge a year later and showed equivalent proficiency in the procedural problems as compared to the traditional students.

Although the body of research described above has examined the teacher's role in classes where argumentation and justification are valued in the development of students' mathematical thinking, and the effect this has on student's understanding, the goal of such classes is not to develop better proof-writers. At many universities, the courses that are designed to teach students how to prove are called "transition courses" and tend to be disguised in other content. Clearly, teaching proof outside of context is unreasonable. However, many of these classes do not focus on the proof writing, but rather on the content with a secondary goal being the improvement of students' proof writing.

In terms of research on transition courses, there is very little if any at all. The research at the undergraduate level described earlier may be from transition courses; however this information is not reported. What is clear is that proof was not the focus and that the role of both the instructor and students during discussions were not studied. The research I conducted is an analysis of the instructor's and students' roles during student-to-student discussions in an inquiry-oriented classroom in a transition course where proof in the context of Number Theory is the content. The ultimate goal of this course was to improve students' proof writing. Thus the research I have conducted has expanded on the study of inquiry-oriented undergraduate classes as well as begin to fill the gap in the research on teaching proof at the undergraduate level.

3.4 Mathematical Discussion

Several of the research studies mentioned in the previous section on inquiry-oriented classes focused on the development of the norm of argumentation or discussion as a part of the analysis of these classrooms. However, none of these studied both the roles of the instructor and students during these discussions.

There is a large body of research on mathematical discussion. Some of this research focuses on whole class discussions while others focus on small group discussions within a mathematics classroom. In addition some discuss the techniques of discourse analysis.

There are several researchers that have looked at discourse in a whole class setting. Edwards (1993) studied a kindergarten class and explored the discourse patterns. He found that these patterns are generally controlled by the teacher and in studying them one can gain insight into the conceptual understandings that he claims are the content of these discussions. Another example of whole class discussions in an elementary classroom comes from the work of Wood, Cobb, & Yackel (1993). In this research Wood et al. compared the discussion and interaction patterns in a traditional mathematics class to those in an inquiry-oriented classroom. They found in the traditional classroom the instructor controlled and directed the interactions and therefore discourse in which genuine communication occurred was rare. They claim that this minimized the opportunities that students had to engage in mathematical activity – clearly linking discourse and mathematics learning. They found that in the inquiry classroom the students were actively engaged in mathematical argumentation.

A third example of discourse in an elementary classroom comes from the work of Cobb, Boufi, McClain, and Whitenack (1997). In their study they claimed that when students participated in discussion they were able to reflect on their own thinking. This, they claim, allowed for mathematical learning to take place. Rather than focusing solely on what the content of the discourse implied about student understanding, these examples looked at the discourse patterns in these classes with the claim that this discourse affects mathematics learning and understanding.

In addition to the research on whole-class discourse at the elementary level, there is some at the middle school level. Knuth & Peressini (2001) studied a seventh grade math class and described the discussions that occurred in this classroom, grouping them by those discussions that conveyed meaning and those that generated meaning. They also claimed that students will gain a deeper understanding of mathematics when they participate in discussions that generate meaning by allowing them to use their own statements as well as those of their peers and teacher as thinking devices.

Another example of research on discussion at the middle-school level was done by Sherin (2002). The focus of this research was more on the role of the instructor and how they facilitated discussions. They found that this particular instructor struggled with balancing their focus between the process of the discourse and the content of the discourse. In this case, rather than study the discourse patterns, Sherin (2002) focused on the role of the instructor throughout the semester. This research has a similar focus as my dissertation research; however I studied the role of the instructor as well as the students during discussions.

Many of the examples of discourse research at the undergraduate level were mentioned in my synthesis of the research on inquiry-oriented classrooms. One example is the work of Yackel (2002) on collective argumentation in an undergraduate classroom. There is also research conducted by Ju and Kwon (2007) that studied the students' use of personal pronouns in discussion and what this implied about their beliefs about mathematics. They found that when students spoke in the third-person they positioned themselves as passive recipients of mathematics and when they spoke in the first-person they positioned themselves as active mathematical learners. They found that in an inquiry-oriented differential equations class the discourse shifted from third person to first person over the course of the semester. They also briefly discuss the instructor's role in promoting this shift.

My research would begin to fill a gap by studying both the roles of the students and instructor in discussions between students in a whole-class setting at the undergraduate level where there is currently less research on whole-class discussions.

Another body of research on discourse looks at discussions in small groups. These tend to focus on what the discourse implies about student understanding. For example, Sfard and Kieran (2001) analyzed discussions between two thirteen-year old boys. They use this analysis to discuss the differences between effective and ineffective communication and claim that ineffective communication does not promote understanding. Doise, Mugny, & Perret-Clermont (1976) conducted a study on individual children to see if discussion with other children allowed them to solve a task better than when they were on their own. They found that these children performed better cognitively when they were involved in the social interaction.

One example where the researchers also were interested in the roles of the participants in the discussion comes from Zack & Graves (2001). They studied small group discussions that the instructor was not a part of and used semantic discourse analysis to identify themes in these discussions as well as to understand the roles of the students in these discussions. They claim, based on their data, that knowledge and understanding are constructed in the context of the mathematical activity and the participants in that activity.

This research differs markedly from my dissertation research which makes no attempt to determine student understanding, rather I claim that the level of student understanding is high in this particular class based on research conducted on similar classes (c.f. Smith, 2006).

The body of research on discourse that I am drawing upon the most in my own research is that which claims a link between discourse and mathematical understanding. Sfard & Kieran (2001) claim that thinking is an act of communication, and thus discourse constitutes thinking. They also stress that communication does not always imply thinking and understanding, but that this communication must be effective. Wood et al. (1993) also claim in their research that psychology states that by discussing mathematics individuals are given the opportunity to reflect on their own thinking. In fact Vygotsky (1987) focused on language as the central mechanism of learning. In the work that Yackel & Cobb (1996) conducted in an elementary classroom they argue that mathematical argumentation provides learning opportunities for both the students and the teacher.

Overall, most research on discussion makes the claim that discussion and mathematics learning and understanding are linked. In fact the NCTM Standards (2000) state that

> Instructional programs from prekindergarten through grade 12 should enable all students to—
>
> - organize and consolidate their mathematical thinking through communication;
> - communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
> - analyze and evaluate the mathematical thinking and strategies of others;
> - use the language of mathematics to express mathematical ideas precisely.

(http://standards.nctm.org/document/chapter7/comm.htm).

Clearly, discussion and communication are vital to mathematics understanding. My research focuses on the roles of the instructor and students in discussions that involve at least two students. This links the research on whole-class discussion to that on small group discussions between students. In addition, I did not attempt to study what these discussions implied about student understanding; rather I studied these discussions in order to gain insight into these roles in the context of an inquiry-oriented classroom where mathematical proof is being learned.

3.5 Conclusions

The literature shows that the role of proof in mathematics as well as mathematics educations is social in nature. In addition the process by which proofs are validated is through acceptance by the mathematical community. This supports the notion that the teaching of proof should focus on the social nature as well as the community norms for determining validity. In addition one of the roles of proof is to explain and bring about understanding. This too is valuable to the teaching of proof.

There is a large body of research that has studied elementary classrooms in which discourse, understanding, community, and argumentation are valued. However, this type of research is lacking at both the secondary and undergraduate level. This is especially true at the undergraduate level; this may stem from the fact that most undergraduate mathematics courses are not taught with an inquiry-oriented method but rather through lecture. Although there is a growing body of research on undergraduate classes that are inquiry-oriented, there is none that studies classes that teach proof as the primary content.

The majority of research done on proof has been of student understanding and reflects a deficit view. In addition, there is very little, if any, research on the teaching of proof in an inquiry-oriented class. By studying the discussions about student-generated proofs at the undergraduate level in an inquiry-oriented class where participation in a collaborative community and understanding are valued, I argue that I am extending the current research on inquiry-oriented classes at the undergraduate level as well as the research on discourse and the teaching of proof. In addition, this research is supported by the research on the role of proof and the validation of proof within the mathematical community. Thus my research fills current gaps, but also extends and logically combines several bodies of existing research.

The relevant literature supports my research question in several ways. The first component is that of discussion and its importance in the classroom of study. The research that supports this comes from the research on inquiry-oriented classrooms as well as the research on discourse. Most of the classrooms in which this research took place put an emphasis on the discussion of ideas, as well as what role the teacher played in developing the norms for discussion and disagreement, or on what the discussions imply about student understanding. In addition, my question is supported by the social nature of proof in two ways. The literature shows that proof is social due to the role of proof in communicating mathematics as well as the social aspect of validating proofs. The research shows that the validity of a proof depends upon the community it is being presented to as well as whether or not it convinces qualified judges. Both of these speak to the social and community aspect of proof. In addition the research on inquiry-oriented classrooms, in particular Goos' s work in classroom communities at the secondary level, support the further study of these types of classrooms at the undergraduate level – in particular in the teaching of proof.

The research on the philosophy of proof emphasizes understanding in the process by which mathematicians determine if a proof is valid. One of the criteria offered by the research is that the proof convinces a group of qualified judges. The idea of needing to convince others supports the idea that writing valid proofs entails writing for understanding. In addition the research on student understanding of proof clearly shows a lack of understanding of mathematical proof. This supports a need to study a classroom in which a deep understanding of proof is valued. The last piece of the research that supports this component of my research question is the research on mathematical discourse. Most of the research on discourse makes the argument that participation in mathematical discourse promotes mathematical understanding. This implies that in order to research a classroom in which understanding is valued we need to study the mathematical discussions that take place.

4. Methodology

Over the course of my dissertation work the data led me in a direction that re-focused my research question onto discussions between students. The focus of my research narrowed to that of student-to-student discussions and the roles of both the instructor and the students in these discussions. I believe that this focus fits within the scope of my original research question. The original research question I proposed to answer was: What is the role of the instructor throughout a semester in developing a classroom community in which discussion, participation in a collaborative community, and understanding are norms and are valued as integral parts of the process of constructing proofs? The following describe in more detail the three main parts of my original research question and how they have been altered to focus on student-to-student discussions.

- *Discussion* – In many of the research studies on inquiry-oriented, student-centered courses argumentation and discussion were key norms that the instructors worked hard to develop. Thus by looking at the role this instructor played in developing discussion throughout the semester, either through the questions he asks, or the moments he chose not to speak, I hoped to better understand the manner in which discussion developed over the semester in a course whose purpose is to promote a more complete understanding of proof.
    - This piece of my original research question was refocused to include the types of participation that students exhibit during different types of discussions with at least two student participants.

- *Participation in a Collaborative Community* – The idea of collaboration as a norm in the entire proof process speaks to the social nature of both the proof construction and validation processes. This is supported by the literature on the role of proof in both mathematics and mathematics education. I hoped to understand how the instructor enabled the students to work together, collaboratively, on creating proofs, establishing the validity of proofs, and altering proofs in order to make them valid.
    - This part of my research question was re-focused to include an interest in how this norm of collaboration varies among the different types of discussions that occur throughout the semester. In particular, I was interested in how the students interacted with each other in different types of discussions and at different points in the semester.
- *Understanding* – The final piece of my question relates to the norm of understanding. In most proof courses at the undergraduate level students leave with an algorithmic understanding, if any, of the proof process (c.f. Recio & Godino, 2001, Segal, 2000, Hart, 1994). However one of the roles of proof is to explain and promote a deep understanding of the mathematics. I hoped to explore how the idea of deep understanding developed in this particular course.
    - This part of my research question fits with my revised focus on discussion. Much of the literature on discussion argues that discourse and understanding are linked. I have addressed the norm of understanding by

better understanding the discussions that students have with each other throughout the course of a semester.

Thus my research question has adapted to: What are the roles of the instructor and students throughout a semester in the discussions that occur in an inquiry-oriented course where the roles of proof to communicate and explain mathematics are valued as integral parts of the proof construction and validation processes?

4.1 Design of Study

The design for this study is modeled after the study conducted by Dr. Jennifer C. Smith and her research team in the fall of 2003. I was a part of this research team and am hoping to further this research. This study also involved a transition course[4], Introduction to Number Theory, and was taught by the same professor, Dr. Stone. We collected video data of every class meeting, conducted interviews with Dr. Stone, and task-based interviews with seven students. During the analysis of the video data from the 2003 study the roles of the instructor, the class, and the student presenter were analyzed based on different themes that emerged from repeatedly studying the video data using open coding techniques. The research I have conducted can be seen as an attempt to expand the work done in the 2003 research with a specific focus on the role of the instructor and the role of the students during discussions involving two or more students.

This study's design is qualitative rather than quantitative or mixed methods. The reason for this choice stems from the nature of my research question. In my study I did not hope (or attempt) to show causation for some result or attempt to test any hypotheses.

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4 At this particular university a transition course is meant to help students learn mathematical proof before continuing into more proof-based mathematics courses. The intention of this course is for it to be taken prior to courses such as Real Analysis and Abstract Algebra.

While these are valid research methods, they do not fit with my research question. I chose to study a complex and detailed issue in depth in order to better understand its complexities. This type of research is done best through qualitative methods (Patton, 2002). Since the research question is focused on the behaviors of the instructor and students in a single course it would have been quite difficult to incorporate quantitative measures that offered anything valuable in their analysis. My research question was best addressed through a method that allowed for rich descriptions of the types of discussions that occurred in this particular class.

This work has also been informed by the perspective of grounded theory, but does not follow all of the canons of grounded theory as proposed by Strauss and Corbin (1990). Grounded theory is a method that allows the researcher to generate theory systematically that is grounded in empirical data in order to develop a theory that would fit the data. Grounded theory tends to limit bias since the researcher is not beginning the process of answering the research questions with a pre-determined theory he or she is trying to test. Rather the researcher first sets out to gather data and then to systematically develop a theory based on what emerges from the data (Walker & Myrick, 2006). This method was developed by Glaser and Strauss and combines two different processes of data analysis. They felt that neither on its own could lead to the development of a theory, rather a combination of both that involves the procedure of systematically coding all the data and then looking for categories and themes in order to develop theory is needed. Strauss and Corbin (1990) state that the procedures used to develop a grounded theory are meant to develop a set of concepts that provide a thorough explanation and description of the object of study. They argue that coding is the fundamental analytic process and is what leads from the data to theory development. This type of research is thus very organic in nature.

One of the canons of grounded theory is that data collection and analysis are interrelated processes. Strauss and Corbin (1990) argue that analysis must begin as soon as data is collected. In the case of my dissertation, this was not the case. The entire semester worth of data was collected prior to analysis. However, the video data allows for the focus of the research to change as analysis is done which is what occurred in this case. I was able to capture most everything that occurred within the class, allowing for my analysis to change based on the data. Thus, although one of the major canons of grounded theory was not followed in my research I was still able to examine all the possible avenues that the data might take me.

As described by Strauss and Corbin (1990) my analysis allows for concepts and common themes to develop and frame the theory that comes from the data itself. By looking for themes rather than focusing solely on the raw data my analysis is reflective of grounded theory. In addition, my concepts and themes are grouped into categories based on the types of discussions occurring, which also follows the canons and procedures of grounded theory.

By using the techniques of grounded theory I will allow for the theory based on my results to develop through the collection and analysis of the data, allowing the data to guide the research in terms of my research question. As can be seen by my choice of a grounded theory approach I was able to adapt the focus of my research based on the themes that emerged in my data.

4.2 Participants

The main participant in this study is Dr. Matthew Stone. Matthew Stone has been teaching for over 30 years and has won numerous teaching awards, although he has never undergone any formal teaching preparation. During his graduate work, Dr. Stone was a student in several courses where a version of the modified Moore method (MMM) was used. He is one of the co-authors of the theorem sets used in the Number Theory course that I observed and taught using these Number Theory notes several times prior to the fall of 2005. In addition he serves as mentor to other professors who are first adapting these notes to their courses. The secondary participants are the students in Dr. Stone's class. There were approximately 24 students registered for the class, although a few of these dropped before the end of the semester. There were approximately 20 students who attended regularly. These students had a mixed background in mathematics and were from a variety of majors. The following table (Table 1) shows the variety of majors represented by the students in this particular course.

<table>
<tr><th colspan="2">Majors Represented</th></tr>
<tr><td>Mathematics (18)
Pure mathematics
Math Sciences
Math for Secondary Teaching</td><td>Other (6)
Computer Science (2)
Government (1)
Chemical Engineering (1)
Aerospace Engineering (1)
Management Information Systems (1)</td></tr>
</table>

Table 1

All had completed the prerequisite of Calculus, and most students scored an A in Calculus (only one student scored a C).

Almost all of the students registered for this particular class based on the Instructor's reputation. This may have had some impact on their roles in the class since they had a prior knowledge of this instructor's manner of teaching and purposefully chose to enroll in the course as a result. On a survey administered the first day of class, as part of a second research study, the students were asked what they thought a mathematical proof was. Many of the students answered with some sort of logical explanation. For example, one student wrote "a series of steps that demonstrate the accuracy of a mathematical statement" and another wrote a "step by step method of using pre-proven equations and strategies to prove new or unproven math problems". Other answers noted the importance of verifying or establishing the truth of a statement and others answered with an explanation that involved the process of explaining one's reasoning in solving a problem. The final question the students were asked on this survey was to rank the roles of proof (five recognized roles from the literature) from most important to least important. These were fairly equally split between the five as the most important.

4.3 Context

The structure of this Number Theory course is quite different from a typical undergraduate mathematics course and that merits attention. This course was taught with a focus on inquiry-oriented learning[5]. In this particular class, the students were given a set of theorems and questions that they were asked to prove or justify. The activity of a class period consisted mainly of the presentation of student solutions to these problems and discussion of the presented proofs. Thus, the students were expected to complete the problems and theorems prior to their proofs being presented in class. Often during the semester other questions or extensions of questions were brought up and class time was spent discussing these as well as presenting solutions to these additional problems. This course also had a teaching assistant[6] who played an active role in the class by participating in discussions, grading the students' homework solutions, and holding office hours. There were two exams during the semester and one final exam. The second exam consisted of a take-home portion, whereas the other two were in-class exams.

4.4 Data Collected

The primary data source for this research is video tapes of class meetings. On average two to three classes per week were video taped. The primary purpose of these tapes was to capture the decisions and actions of the instructor. The secondary purpose was to capture the actions of the students throughout the class period. Thus, these videos are mainly focused on the instructor. Although the camera focused on the instructor for the majority of the class it was set up on the opposite side of the classroom and thus also captured almost half of the students. However, often when a student was talking, the student was not visible on the video. For instance, when a student presentation was being

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5 Often this course is described as a version of the modified Moore method which was described in the Introduction. I am choosing to refer to the method of this course as inquiry-oriented learning because, as I described earlier, it is a form of this type of learning. In fact the instructor rarely refers to it as the modified Moore method.

6 I was not the teaching assistant during the semester of study, although I served as a teaching assistant for this course in past semesters.

given and the instructor moved to the back of the room, the camera followed him rather than the student presenting. If the instructor went to the front of the room the camera followed him no matter who was talking. In addition, the camera occasionally panned over the entire class in order to capture the atmosphere of the class at a particular moment. The data collection was completed in the fall of 2005.

This choice for data collection supports my research questions because the videotapes provide a way of capturing the actions of the professor and students during most class days. By using video, rather than just field notes, I was able to return to the original circumstances throughout the entire analysis. Thus, this data will limit bias as well as provide multiple views of the discussions that took place over the course of the semester.

In addition to the videotape data, I conducted interviews with the instructor and six students I selected to serve as supplemental data should it become relevant in the analysis of the video data. The interviews with the instructor occurred at the beginning and the end of the semester. The interview at the beginning of the semester attempted to capture the instructor's view of this class in general, how he teaches, what his role is, and how this changes over time. The final interview invited the instructor to reflect on the semester, his view of the role he played, and if and how that role changed over time. The students were interviewed individually about their impressions of the class and the instructor several times throughout the semester. The protocols for each interview are included in the appendix.

The last type of data I collected was audio taped guided reflections. The guided reflections occurred as often as they could be scheduled, on average every other week. During these reflections, I asked the instructor to tell me about the class and what he thought was interesting about what was occurring. These reflections tended to become discussions about things that had happened in class, how he had handled them, and why he had chosen to do so in the manner he did. Also, these reflections involved his decisions regarding the tests in the class and how these decisions were impacted by his goals for the class.

During the course of the video analysis the focus of my research question shifted. I found that the data collected from the other sources besides the videotapes were not useful in my analysis.

4.5 Initial Analysis

The initial stage of my analysis involved watching each video tape and writing a detailed description[7] of the entire class session. I broke the class sessions into meaningful pieces based on changes in content, type of activities, as well as changes in the focus of the discussions or activities. After completing the descriptions of each video I began the process of open coding.

<u>4.5.1 Open Coding</u>

During the first round of open coding I chose to focus my attention on the first theme of my original research question – discussion. In doing so I generated a list of instructor and student behaviors (or possible codes) that occurred throughout all parts of

7 These detailed descriptions were not transcriptions; however they included transcribed portions of the discussions. In addition these descriptions incorporated what the instructor and students were doing in addition to what was being said.

the discussion process. These included what occurred when initiating discussions, participating in discussions, ending discussions and any instance I felt was relevant to developing the norm of discussion. After organizing these codes into meaningful segments I found that there were far too many behaviors from far too many types of situations for analysis to be meaningful[8].

After going through all of the video data I found that discussions involving students were particularly interesting. I decided to code my data again looking for instances of discussions involving at least three people and at least three separate utterances. I chose to divide discussions based on a distinct change in content or a loss of a participant. For example, if there are three (or more) people involved in a discussion then Dr. Stone interjects and uses back-and-forth questioning with a single student, the discussion has ended until a third person re-enters the discussion. For example, the following transcript from 10/26/2005 begins with the student R[9] giving a presentation of how to explain a calculation of 2^{20} mod 41. The instructor is referred to as *M*.

> **R:** Okay so, in this one just walks us through four different steps and says why is each step true. The first one, and this is what we're trying to show. (Mike moves towards the back of the room). This step number uh, one is two to the fifth equals negative 9 mod 41. And if you plug that into the definition that turns out to be saying that 41 divides 41 and that's true. (Pause)
>
> ***M:*** All right.
>
> **R:** and step two uses theorem 1.15 where we can raise each side to a certain power and we raise both sides to the fourth power. And let's see, step three um. (Pause). Oh, this is just saying that 81 is equal to -1 in mod 41 because you go, you start at 41 and go around, well you go up to 82

8 This list of initial codes can be found in the Appendix.
9 In the transcripts all students are referred to by their firs t (or first and second) initial to maintain anonymity.

well that's one less so that's -1. And then we use the, the, theorem 1.15 again to square both sides.

O: Could you explain step three again.

L: Yeah, so I think you explained the 81 squared is congruent to -1 squared, but what about the first part, the two to the twenty.

At this point the discussion has begun (based on my definition of three participants). In this case the three are R, O, and L.

R: This is equal to this right here. (Pause).

O: Yeah but why is it congruent to -1.

R: Why is this congruent to negative 1?

B: I think you're just missing stuff, all you have to do is say that two, just make the first part, you're bypassing the 2 to the twentieth is congruent to 81 squared. Could you just explain that? You know what I'm saying? You're just missing that one step.

O: Try to –

L: It's just that, yeah, negative nine, 81 squared equals -9 squared squared.

O: Yeah, well I believe that, but how did he get to the negative one.

S1: In other words, if I'm hearing you correctly –

L: --Then that's the one.

S1: Um, the 81 mod 41 equal to -1 mod 41, is that what the question is?

O: Yeah, that's the part that I don't know how he came to it.

S: And then he didn't write down the part about the two to the twenty congruent to 81 squared.

L: Well that's because eighty, 41 times 2 would be wait what.

S2: eighty-two

L: eighty-two.

Ma: 41 divides eighty-two so 81 plus one is divisible by 41. So eighty-one is congruent to negative one mod 41.

L: Yeah, I think you need that. Since 82 is congruent to 0 mod 41, then 81 would be congruent to -1 and then you can square both sides.

R: I showed that here.

O: Well (trails off)

(Pause)

M: Okay, so are we clear on this? I don't. R are you clear?

R: Yeah.

M: So what did they want you to say here?

R: Just right in here, that excuse me, that um, 82 is congruent to 0 mod 41 so then 81 is congruent to -1 mod 41.

M: Okay.

R: And then part 4 is just –

The final interchange between the instructor, *M*, and the presenter, R, is the end of the discussion since there are no longer at least three participants in the exchange.

During this second stage of open coding I focused just on discussions based on my definition of discussion as an instance of three or more people speaking with each other. As I continued my open coding this definition of discussion evolved in order to focus the data analysis on the circumstances that would best answer my research question. I will give the precise definition after describing the entire initial coding process.

After identifying what was considered a discussion, I recorded who initiated the discussion, the names of the people involved including how many times they spoke, and the description of the discussion. For example, in the discussion from 10/26 regarding the proof of 2^{20} mod 41, I would have coded the discussion in the following manner:

Date	Time	Initiated By?	Participants	# of Utterances
10/26	09:15	O	O R L B S1 S2 S Ma M	6 5 6 1 2 1 1 1 3

Table 2

While focusing on these particular aspects of the discussions I found that Dr. Stone was involved in many more discussions than I expected, but what seemed to change was his role in the discussion. He sometimes focused (or re-focused) the conversation, sometimes asked questions or made statements, and other times he played the role of a peer participating in the discussion equally with the students. So rather than be concerned how much he talked, I chose to focus on the type of talking he did. In addition, I found that I needed to not only focus on who was involved in the discussions, but how they were participating. Were they responding to one another? Were they resolving the issue raised, or relying on Dr. Stone to resolve the issue?

I also noticed that there were a few student-to-student interactions that only contained two students and did not qualify as a discussion under my definition, but were obviously relevant to the idea of students talking to students. In addition, there were several discussions that fit my definition; however they included Dr. Stone, the TA, and a single student which did not fit the focus of student-to-student interactions I was interested in.

After the second round of open coding I chose to make changes to my definition of discussion as well as expand my themes of interest. I revised my definition of discussion to include at least two student participants as well as at least three separate utterances. In addition, I chose to add three more themes "role of Dr. Stone", "role of students" and "how discussion ended", three more components of interest based on my analysis thus far. These allowed me to focus more on the way that both Dr. Stone and the students participated in these discussions. In addition the revised definition removed the discussions only involving a single student and Dr. Stone (and/or the TA) from my analysis.

I returned to my data for the third round of open coding where I identified behaviors for each of the three themes I mentioned previously. During this round of coding I found that my descriptions of the role of Dr. Stone did not truly capture his role. I used descriptions such as "comment on student proof", "ask for questions", "answer student questions", etc. This did not truly capture the role he played; it merely captured his actions. For example, I was interested in whether he played the role of the authority by giving the "right" information, or the role of a guide scaffolding or leading the discussion in any particular direction, or did he "learn" along with the students, trying to understand the proof presented and asking questions that related to that goal. At this point I re-focused my descriptions of Dr. Stone's behaviors in order to better capture his role in the discussions and used the coding scheme from the 2003 research study in order to aid in this process. Several of the behaviors identified from the 2003 data were added to my set of descriptions as well as others that I noticed in my data and found relevant.

After the third round of coding I developed my initial coding scheme. I developed this coding scheme based on the main themes I found during my first three runs through my data. After the first run, I decided to focus on student-to-student discussions (those that included two or more students). I also decided that besides knowing who was involved in each discussion and how many utterances they each had I was interested in who initiated the discussion – this resulted in the first column of my coding scheme. After going through my data the second time I decided that the other three columns were themes of interest – how did the conversation end, what role each student played, and what role Dr. Stone played. During my third run through the data I felt as though my descriptions of these were not very useful. I felt that I wasn't capturing what the students were saying and what Dr. Stone was saying, just that they were saying something – either answering a question, making a comment, or asking a question. I revised the codes I had used during the second round of coding and added others in order to develop my initial coding scheme.

4.5.2 Developing Coding Scheme

In the next step of my analysis I took my initial coding scheme and started coding random discussions to see if these codes truly captured what is in the data. During this stage I continued to add codes to my coding scheme as well as revise codes. These additions and revisions were based on new behaviors I found in the data that I was unable to code using the initial coding scheme. For example, while coding some select sections

I decided I wanted to keep track of how many of the student comments were not related to a previous student comment due to what this might infer about the type and level of interaction between students during the discussions I was coding. In addition, I found that the TA played an active role in some discussions and thus needed to develop codes to capture his role.

After this random coding I added a column for the role of the TA using many of the codes that were in the role of Dr. Stone column, but not all. In addition I added several more codes for Dr. Stone including interrupting a proof and asking a question of the presenter in the case of an incorrect proof as well as a correct proof. I also reorganized the coding scheme into two rows. One row represented the behaviors that support the students as the main focus of discussions and the other row represented behaviors that tend to focus more on the instructors or a single student as the focus of the discussion.

After these changes were made to the coding scheme, I continued to code random discussions until I was able to code 15 discussions (5 from September, 5 from October, and 5 from November) without needing to revise the coding scheme[10]. The process I went through in developing my final coding scheme is outlined in Figure 1 below.

10 The final coding scheme can be found in Appendix I.

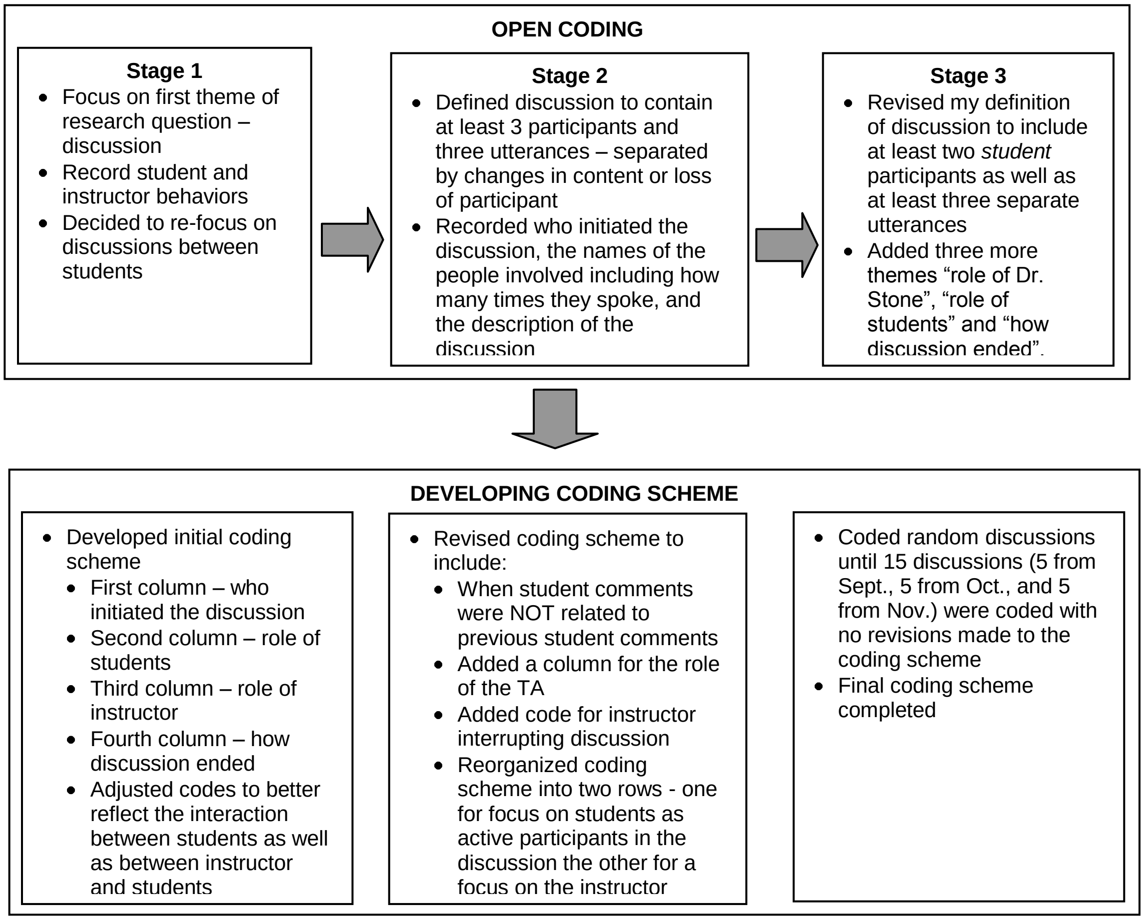

Figure 1

During the final stage of coding I coded every discussion that fit my definition of at least two student participants and three separate utterances. This information is organized in a spreadsheet in Microsoft Excel.

4.5.3 Reliability

The final aspect of coding was to verify the reliability of my coding. I chose an approximately random sample of 10 discussions to re-code using the finalized coding scheme. After coding these 10 discussions I compared this secondary coding to the initial coding I had determined for these discussions. In the ten discussions, 37.3% of all the codes were in disagreement between the initial and secondary coding. This discrepancy of the codes was a result of codes being mismatched or completely left out. All discrepancies were addressed and adjusted resulting in a “final” coding. However, there were a larger proportion of discrepancies during the month of September than any of the other months. The percent of problem codes rose to 44.1% versus 32.8% for the rest of the semester. This is likely due to the unfamiliarity with the coding scheme during the early stages which adjusted itself later in the coding process leading to fewer discrepancies in the later months. Due to these discrepancies I re-coded the entire month of September.

In order to check the reliability again I chose a simple random sample[11] of 20 discussions. After these discussions were coded I compared this secondary set of codes against the “final” codes for the same discussions. While the percent of codes that were in disagreement was still fairly high, 28.7%, the percentage decreased by almost 10% from the first reliability test. Of even more importance was that approximately one-fourth of these problem codes were minor issues such as the distinction between a “student asks a question” and “student asks a question of another student” or whether or not a student was making a comment or responding to another student. The discrepancies in these problem codes were addressed and all changes were added to the final codes. Additionally, some of the same kind of problems relating to the difference between codes

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11 The simple random sample was chosen by first numbering all 168 discussions and then using a random number generator to choose the 20 discussions.

occurred more than once. I ran a search of these particular problem codes and revised all discrepancies related to these codes in all of the data.

4.5.4 Whole Discussion Codes

In order to organize these discussions and begin to analyze them I came up with four different discussion codes. These codes are meant to categorize the overall discussion and are roughly based on Knuth & Perresini's (2001) descriptions of *univocal* and *dialogic* discussions. I have come up with a continuum based on the levels of student interaction, from discussions that contain very low student interaction to discussions that involve much higher student interaction. Most of these descriptions limit what is said regarding what roles the instructor and students played so that the data could provide more detailed information on these roles in the different types of discussions throughout the semester. Thus the data was used to create rich descriptions of the types of discussions that take place in an undergraduate transition to proof course taught using inquiry-oriented learning.

The four codes I have created are *univocal*, *Socratic, student-centered*, and *dialogic.* In a *univocal* discussion the instructor remains as the authority in terms of addressing students' comments and questions. In addition there is little to no give-and-take either between the students or between the students and instructor. Overall these are very brief discussions. A *Socratic* discussion involves some more give and take; however this is limited to interactions between the instructor and the students. In general these are question and answer sessions that often involve scaffolding of ideas in the direction the instructor envisions. In a *student-centered* discussion the students interact with each other, although this interaction is limited. These interactions don't involve a lot of give and take. Generally these are situations where a student makes a comment or asks a question, another student responds, and the first student follows up with a brief comment. The final type of discussion, which involves the highest amount of student interaction, is the *dialogic* discussion. In this type of discussion there is a lot of give and take between multiple students. The dialogue generally allows for more complex issues to be resolved through a process of multiple questions and responses between students. The continuum I have developed for these four discussions is illustrated below in Figure 2.

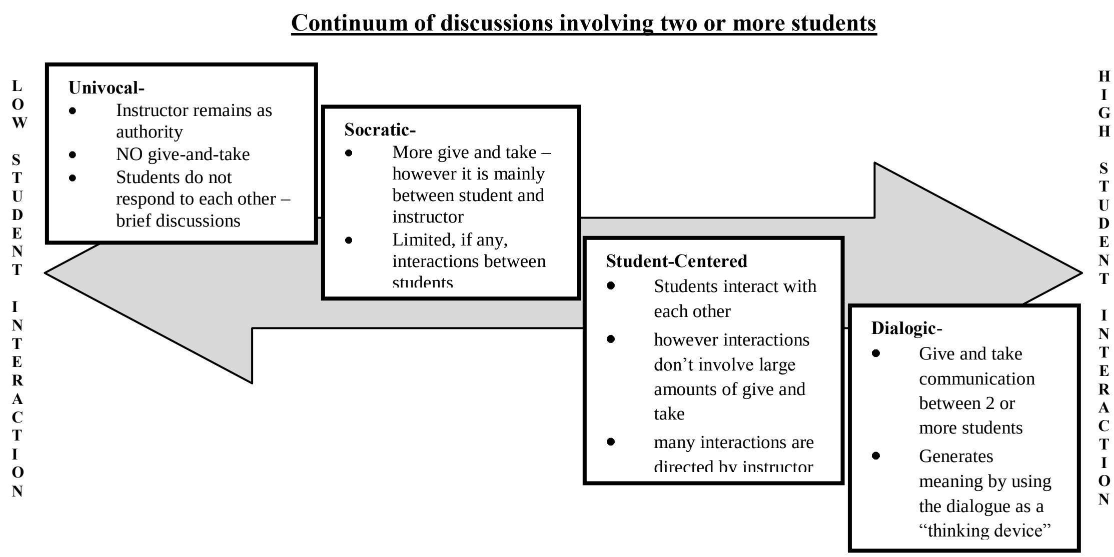

Figure 2

While coding using the discussion continuum I found that several discussions had characteristics of two adjacent codes. These discussions were coded with both discussion codes that were relevant. After using this continuum of discussion codes to code every discussion in my data I once again checked the reliability of my coding. I chose a simple random sample of 20 discussions and re-coded just the overall discussion codes. Of the

20 discussions re-coded only 5 differed from the original codes. Four of these five discussions were either coded as two different discussions originally but as a single discussion in the re-coding or vice versa. These four codes were revised and changes were made to the final codes. The fifth discrepancy was initially coded as *student-centered* and re-coded as *dialogic*. These are two adjacent codes and the discrepancy was resolved in the final coding.

4.5.5 Difficulty of Content

During the course of my analysis I decided to examine the difficulty of the content associated with each of the three main discussion types. I was curious to see if there was any relationship between the type of discussions that the students were engaged in and the difficulty of the content that they were discussing. Based on my experience with the course, as well as input from instructors who have taught using the same structure and notes that were used in this class, I was able to rank all the theorems from the notes as low, medium, or high content. I used this information to code the difficulty of the content of all the student-to-student discussions.

5. Results and Analysis

After the coding was completed, including the reliability stage, I first wanted to look for anything of interest related to the number of times all of the student-to-student discussions occurred and how this varied over the course of the semester. I grouped the class days that were videotaped into three equal groups (each with 8 days): September, October, and November/December. After counting the number of discussions in each of these time periods, I found that the total number of student-to-student discussions did not vary dramatically over the course of the semester. In the first third of the semester there were 52 of these discussions. This total was the same for the second third of the semester and rose to 64 discussions in the final third of the semester.

I then took the counts of each of the four main discussion types, *dialogic*, *student-centered*, *Socratic*, and *univocal* from each of the three thirds of the semester. This data is displayed in the graph below (Figure 3):

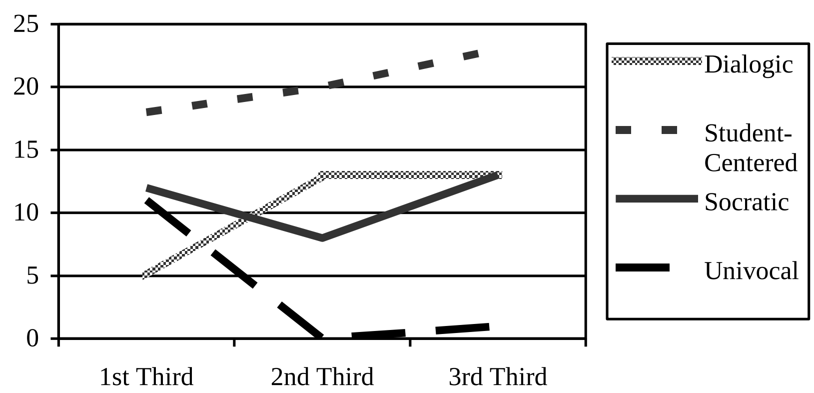

Figure 3

We can see from this graph that both the *dialogic* and *student-centered* discussions increased in terms of total count as the semester progressed. In contrast, the *Socratic* and *univocal* (both the low student interaction discussions) declined over the course of the semester. However this may not be an accurate picture of the distribution of each type of discussion in terms of the total number of discussions for a particular part of the semester. The following graph displays the proportion of all discussions each discussion represented from each third of the semester:

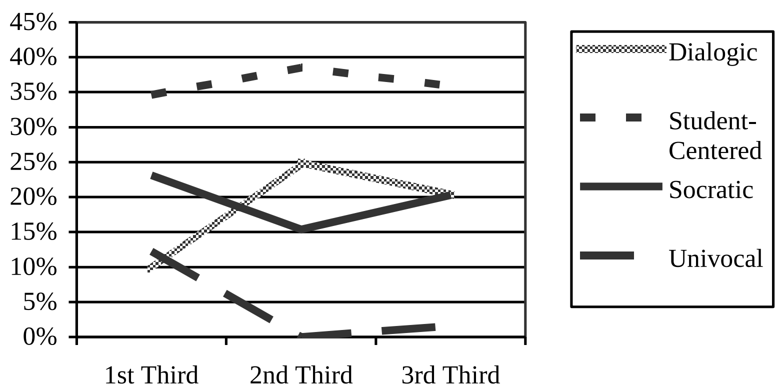

Figure 4

When we look at the percents we find that *dialogic* discussions actually decreased from the second third of the semester to the third, in terms of the proportion of all discussions they represented. We also see a slight drop in the proportion of *student-centered* discussions in the third third of the semester. This also means there was an increase in the proportion of *Socratic* and *univocal* discussions at the end of the semester.

Next I decided to get an overall picture of who initiated the discussions. I found that this tended to be Dr. Stone, or the TA. However, the percentage of discussions initiated by the instructor or TA was approximately 65% in the beginning of the semester and decreased to approximately 55% by the end of the semester, with a large dip in November.

The final overall piece of the data that was of interest was the link to the difficulty of the content. After coding each discussion for the difficulty of the content being discussed I calculated what proportion of each of the discussion types were high, medium, and low content. This data is summarized in the graph below (Figure 5):

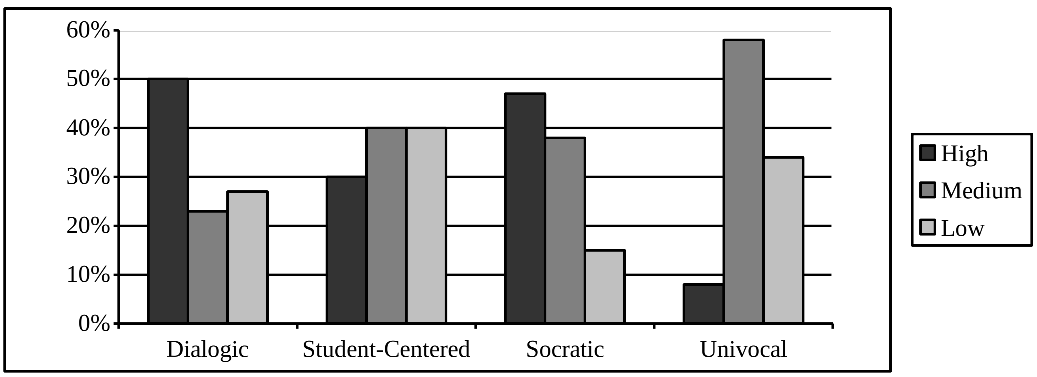

Figure 5

We can see from the graph that the proportion of *dialogic* discussions that were regarding high content areas is twice as high as the proportion of medium or low content area *dialogic* discussions.

I then changed my focus to a comparison of the discussion codes in an attempt to better understand the types of discussions that occurred in this inquiry-oriented course. The discussion code that occurred the most was the *student-centered* code, which appeared 60 times, almost twice as much as the next most frequent code. The next two closest discussion codes (in terms of quantity of appearance in the data) were *Socratic* and *dialogic*, with 33 and 31 discussions respectively. Although the *Socratic* codes can be considered to have low student to student interaction, it is worth exploring the large number of discussions that can be considered to have high student to student interaction.

By taking the discussion codes and grouping them into low, moderate, and high student interaction some interesting observations can be made. In the low student interaction group are the *univocal*, *Socratic/univocal*, and *Socratic* discussions. In the moderate student interaction group are the *Socratic/student-centered* discussions. In the high student interaction group are the *student-centered*, *dialogic/student-centered*, and *dialogic* discussions. Overall, almost 60% of the discussions coded had high student interaction. Thus, when two or more students participated in a discussion approximately 60% of the time there was a high level of interaction between them. The moderate student interaction discussions made up about 10% of the discussions leaving low student interaction discussions to make up about 30% of the discussions.

The fact that 30% of the time in discussions involving two or more students the students had a low level of interaction between them is interesting. I was curious if this varied throughout the semester. First I split the data into 3 equal groups, 8 classes from the beginning of the semester, 8 from the middle, and 8 from the end. I found that during the first third 44.2% of the discussions had low student interaction and that this number decreased later in the semester. During the middle of the semester this number dropped dramatically, with only 15.4% of the discussions having low student interaction. At the end of the semester this number rose again, however was only 28.1% of all discussions. In contrast, the percent of high student interaction as well as moderate student interaction discussions both grew dramatically by the middle of the semester. The high student interaction discussions grew in percentage of the total from 46.2% to 75% and the moderate student interaction discussions grew in percentage of the total from 5.8% to 11.5%. This is an average percent increase of 80.3%. During the final part of the semester both these percentages dropped, however high student interaction discussions only dropped to 59.4% of all the discussions.

I also looked at the average number of participants and average number of utterances. These data are presented in the table below.

	Dialogic	***Dialogic/ Student-Centered***	***Student-Centered***	***Student-Centered/ Socratic***	***Socratic***	***Socratic/ Univocal***	***Univocal***
Average Number of Participants	4.9	6.4	4.2	4.7	4.5	5	3
Average Number of Utterances	12.8	20.6	7.4	11.9	13.6	13.3	4.9

Table 3

This data does not seem to follow any trends. The number of participants per discussion does not vary dramatically, although the lowest number, 3, occurred in the univocal discussions. In addition the *univocal* discussions contained the least number of utterances per discussion, while the *dialogic/student-centered* discussions had the highest (as well as the highest number of participants).

When I developed my coding scheme there was a column for the role of the TA. During most of the discussions[12] that were selected for analysis there were not a lot of TA

12 In the *Socratic* discussions there were 26 TA codes. However, this was dramatically higher than any other discussion, so I chose not to include these with the instructor codes during my analysis of each discussion.

codes and therefore I left them out of the analysis of each type of discussion. However, I will briefly present this data here.

	Dialogic	*Dialogic/ Student-Centered*	*Student-Centered*	*Student-Centered/ Socratic*	*Socratic*	*Socratic/ Univocal*	*Univocal*	*TOTAL*
Focus on students as active participants	1	3	2	5	21	0	1	33
Focus on instructor	4	2	4	4	7	0	0	21

Table 4

Of particular interest is the distribution between those utterances which had a focus on the students as active participants versus those with a focus on the instructor in the *dialogic* discussions. This is the only type of discussion where the TA made more utterances that had a focus on the instructor. Also of interest is the number of utterances made in the *Socratic* discussions that had a focus on the students as active participants (although the total number fits the description of these discussions). The TA made three times as many utterances with this focus versus a focus on the instructor.

5.1 Dialogic Discussions

After studying the distribution of these three groups of discussions I was interested in gaining a better sense of the seven types of discussions.   I began with the *dialogic* discussions.

<u>5.1.1 Role of the Students</u>

I began my analysis with the role of the students in these discussions. I looked for codes I considered high student-interaction codes. These were codes that involved students responding to students, questioning students, directing the discussion, and making follow-up comments or questions. I compared the number of occurrences of these codes versus all other main discussion codes (other than the initiating and end of conversation codes) and found that in *dialogic* discussions 61.0% of the codes fell into this category. This fits what I would expect from these discussions that are coded based on the high level of interaction between students. In particular, the codes regarding follow-up comments made up 29.7% of these codes. That means almost one-third of the time that students were actively interacting they were making follow-up comments and questions, not just making a comment or asking a question and removing themselves as active vocal participants in the discussion.

An additional code of interest related to the role of the students in *dialogic* discussions is the SN code (question/comment is not related to previous student comment). In the 31 discussions there were 7 different instances of students making comments unrelated to previous student comments, but all seven came from different discussions and only accounted for 2.1% of all student main discussion codes.

The only other main discussion codes that are relevant to the role of the students are things like students making comments (SC), responding to the instructor (SRI), and asking a question (of whom is unclear) (SQ). When I developed the coding scheme all codes were put into two categories: codes that focus on students being active participants in the discussion and codes that focus on the instructors. The only code of the three that fits into the codes that focus on the instructors is the SRI code. In the 31 discussions there were 13 instances of this code. Only once was there more than one SRI code within a single discussion. These codes accounted for only 3.8% of all student main discussion codes. Of additional interest are the 119 SRS (student responds to student) codes, 35.1% of all the student main discussion codes, versus the 13 SRI codes.

5.1.2 Role of the Instructor

Next I went on to analyze the main discussion codes representing the role of the instructor in the *dialogic* discussions. Overall there were very few instructor codes represented in the main discussion codes. The instructor codes represented only 11.1% of these codes. I find this a little surprising. During the transcribing of the data it seemed as though the instructor was a very active participant in class discussions involving two or more students. However, it would seem from this data that when there were high levels of student to student interaction, the instructor's role appears to be limited.

Next I took these instructor codes and split them into the two categories described previously: focus on students being active participants in the discussion and focus on instructor. The codes that focused on the instructor made up 44.2% of the instructor codes. These 19 codes included seven PF codes (the instructor facilitates the discussion by calling on particular students). This seems a bit high for discussions with high student to student interaction. In addition there were three PA codes (the instructor answers a student's question or addresses a students' comment directly – acts as an authority), five PCM codes (instructor makes a comment) – two of which followed directly after a PI code (instructor interrupts the discussion or proof presentation). There was one instance of a PSC code (instructor scaffolds a student through a presentation or discussion by repeated questioning) – however the scaffolding was brief.

This leaves us with 25 codes (or 55.8% of the instructor main discussion codes) that related to a focus on students being active participants in the discussion. These codes consisted of seven PR codes (instructor rephrases a students comment or question). These codes were evenly distributed in terms of their position within a single discussion. Some occurred at the end of a discussion, others at the beginning of a discussion followed by a question, and a few occurred in the middle of a discussion. This tells us that a little less than one-sixth of the time that the instructor participated in the discussion involved him rephrasing the ideas of the students.

In two of the 31 discussions the instructor redirected a student question/comment to another student or the presenter. One of these discussions contained two instances of this. These codes show the instructor, possibly, avoiding acting as an authority or controlling which students were or were not participating in the discussion. It is of interest that this only occurred in two of the 31 dialogic discussions.

There were four PQC codes (instructor asks for questions or comments) and all but one occurred at the very beginning of the discussion that contained little to no other instructor verbal participation. There were six PQ codes (instructor asks student or presenter a question) which played a similar role as the PQC codes and mainly occurred at the beginning of a discussion. There were only two PC codes (instructor asks student to clarify or repeat their comment/question) which seems to possibly imply that either students are being more clear in how they express their comments or questions, or other students are able to clarify without being prompted, or possibly that these clarifications and repeats are the instructor's way of making sure students didn't miss a valuable addition to the discussion. It is interesting that there were more than three times as many PR codes (instructor rephrases a students comment or question) as PC codes in a discussion that is characterized by high student interaction. I would expect the instructor to rely more on students clarifying their own ideas than rephrasing them himself.

Also of interest in the *dialogic* discussions was the lack of codes representing the instructor playing the role of a peer in the discussion. In the 31 *dialogic* discussions there were only 2 PP codes (instructor acts as a peer, contributing equally in the discussion). I find this surprising, however it may be that the more the instructor participates (even as a peer) in the discussion the less student interaction there is resulting in an overall discussion code with lower student to student interaction.

5.1.3 Initiating Discussions

In terms of initiating discussions, 21 of the 31 discussions were initiated by students – either the presenter or a non-presenter. In the ten discussions initiated by Dr. Stone, three began with him asking for questions or comments, three began with him calling on a student with a comment or question, three began with him asking a question of the class, and the final discussion initiated by Dr. Stone began with him rephrasing a student's comment and asking a question. This final discussion that was initiated by Dr. Stone also included one of the few PP (instructor acts as a peer) codes as part of the initiation.

5.1.4 End of Discussions

The codes from the end of the *dialogic* discussions were similarly split between a focus on the instructor and the focus on the students as active participants as the initiating discussion codes were split between instructor and student initiators. There were 10 discussions that ended in a manner consistent with a focus on the instructor. These included three CL codes (instructor lectures or makes a statement), one CMS code (instructor gives summary of proof – not an interruption), three CMSS codes (instructor interrupts the discussion and begins a discussion with a single student), and three CMR codes (instructor interrupts a discussion to rephrase the arguments given).

This leaves 21 discussions that ended in a manner consistent with a focus on the students as active participants in the discussion. These included eleven CS codes (issue is resolved by the students and Instructor /TA responds to this), six CST codes (issue is resolved by students), two CML codes (Instructor interrupts a discussion that contains limited student participation to rephrase the issue/comments being made), and two CMRD codes (Instructor interrupts the discussion to redirect the class). The large number of codes referring to the issue being resolved by the students offers evidence that in these discussions, where students have high levels of participation, the students seem to resolve the issues that have warranted a discussion.

5.2 Dialogic/Student-Centered Discussions

I began my analysis of *dialogic/student-centered* discussions in the same way as I did the analysis of the *dialogic* discussions.

5.2.1 Role of Students

I looked for all high student interaction codes; those that involved students responding to each other, questioning each other, directing the discussion, and making follow-up comments or questions. In contrast to the *dialogic* discussions these codes only accounted for 31.8% of all main discussion codes. This is about half of what I found with *dialogic* discussions. I am somewhat surprised since the two types of discussions are adjacent on the continuum in Figure 2. However, there was a strong similarity when considering what percent of these high student interaction codes were the SF code (student makes a follow-up question/comment after previous question/comment). In these discussions 27.9% of all the high student interaction codes were students making follow-up questions. This is only 1.8% lower than in the *dialogic* discussions. So although there were fewer SF codes in *dialogic/student-centered* discussions the frequency with which they occurred in the high student interaction codes was comparable to the *dialogic* discussions.

There were a high number of SC codes (student makes comment) and SQ codes (student asks question – unknown to whom) in these discussions. These codes made up 27.1% of all the main discussion codes. This is about 4% less than the proportion of high student interaction codes – although these do reflect codes that focus on active student participation in the discussion, even if they tend to imply less interaction.

As I did with the *dialogic* discussions, I counted the number of SN codes (question/comment is not related to previous student comment). Out of 9 conversations there were three SN codes. However, there were no discussions that contained more than one SN code. This means in one-third of *dialogic/student-centered* discussions a student made a single comment or asked a single question that was unrelated to a previous student comment. In *dialogic* discussions this was less than one-fourth of the time.

The final codes relating to the role of the student that are of interest included the SRI code (student responds to question/comment of instructor). There were 18 occurrences of this code which were found in just 6 of the 9 discussions. That is an average of three per discussion when at least one occurred. This appears quite high but may also indicate a more active role in these discussions by the instructor. Although there were 18 SRI codes, there were 35 SRS codes (student responds to question/comment of another student). This is almost twice as many. In comparison there were 9 times as many SRS codes as SRI codes in the *dialogic* discussions.

5.2.2 Role of Instructor

All the instructor main discussion codes only accounted for 27.6 % of all the main discussion codes. This is approximately 4% less than just the high interaction student main discussion codes. But it is an increase from 16.5% in the *dialogic* discussions.

I took the instructor codes and once again split these into the two groups, mentioned previously, based on the focus of the codes on students as active participants rather than on the instructor. The codes with a focus on the instructor accounted for 47.2% of all instructor main discussion codes. This is up from 44.2% in the *dialogic* discussions. It would seem at this point in the analysis that as we move away from *dialogic* discussions that consisted of the highest level of student-to-student interaction we find the instructor's role tends to focus less on the students as active participants in the discussion.

These 23 codes included 8 PF codes (the instructor facilitates the discussion by calling on particular students). This is consistent with what I found in the *dialogic* discussions. In addition there were three PA codes (instructor answers a student's question or addresses a students' comment directly – acts as an authority) which is 1.3% lower than the percent of these codes in the instructor main discussion codes in the *dialogic* discussions. However, in both types of discussions these numbers are low.

There were twelve PCM codes (instructor makes a comment). This is almost half of all the instructor main discussion codes with a focus on the instructor whereas in the *dialogic* discussions these codes were only one-fourth of all the instructor main discussion codes with a focus on the instructor. There was only one PS code (summarizes or rephrases the presented proof) and one PI code (instructor interrupts discussion or proof presentation).

The other 20 codes (or 52.8% of the instructor codes) were related to behavior consistent with a focus on the students as active participants in the discussion. These included one PR code (rephrases a students comment or question) and two PRP codes (redirects a student question/comment to another student or presenter) coming from different discussions. There were also two of the PRP codes in the *dialogic* discussions; however that was out of 31 total discussions.

The instructor asked the class for questions or comments in five of the nine discussions. In three of these five discussions this occurred at the beginning of the discussion, in one it occurred once at the beginning and once at the end, and in the fifth discussion this occurred at the end of the discussion. There were 8 PQ codes (asks student or presenter a question) and two PFQ codes (asks a follow-up question based on a student response). There were six PC codes (asks student to clarify or repeat their comment/question) which accounts for almost 20% of this group of instructor main discussion codes. This is much higher than in the *dialogic* discussions.

The final instructor code that is related to a focus on students as active participants in the discussion is the PP code (acts as a peer, contributing equally in the discussion). This code occurred three times, once more than in the *dialogic* discussions even though there was less than a third the total number of *dialogic/student-centered* discussions than the total number of *dialogic* discussions.

5.2.3 Initiating Discussions

Unlike the *dialogic* discussions, a high number of *dialogic/student-centered* discussions were initiated by the instructor. Six of the nine discussions were initiated by the instructor, however most of these involved asking for comments and questions or asking a student to repeat a comment they made. In two of the three discussions that were initiated by students the instructor only had one code during the discussion: PF (facilitates the discussion by calling on particular students). In the third, the instructor had no codes during the discussion.

5.2.4 End of Discussions

The last piece of data I looked at for the *dialogic/student-centered* discussions was how the 9 discussions ended. Although six of the nine discussions were initiated by the instructor, six of the nine discussions ended in a manner consistent with a focus on the students as active participants. Five of the six were coded CS (issue is resolved by the students and Instructor /TA responds to this) which is a much higher percentage of CS codes than in the *dialogic* discussions. In contrast to the almost 30% of the discussions that ended with a focus on the students that were coded with a CST code (issue is resolved by students) only one of the six *dialogic/student-centered* discussions ended in this way. This continues to support the notion that in discussions with high student to student interaction the students tend to resolve the issues raised in the discussions, even if the instructor responds to this more as we move away from high student-to-student interaction discussions.

The three discussions that were ended by the instructor included two with CMSS codes (instructor interrupts the discussion and begins a discussion with a single student) and one with a CMS code (gives summary of proof – not an interruption). In the *dialogic* discussions less than one-third of the discussions which ended in an instructor focused manner ended with a discussion between the instructor and a single student as opposed to the two-thirds of these types of endings in the *dialogic/student-centered* discussions.

5.3 Student-Centered Discussions

I continued my analysis with the *student-centered* discussions in the same way as I did the other analyses – I looked for all high student interaction main discussion codes.

5.3.1 Role of the Students

There was still a lower percentage of all main discussion codes that were high student interaction codes than there were in the *dialogic* discussions, only 45.4%, but this was higher than in the *dialogic/student-centered* discussions. The trend continued in a downward direction in terms of what percent of these high student interaction codes were the SF code (student makes a follow-up question/comment after previous question/comment). In the *student-centered* discussions 22.8% of all the codes related to high student interaction were students making follow-up questions. While this is only 5.1 percentage points lower than in the *dialogic/student-centered* discussions, it is an 18.3% decrease. It is a decrease of 6.9 percentage points form the *dialogic* discussions, or a 23.2% decrease.

There were a little more than half as many SC codes (student makes comment) and SQ codes (**s**tudent asks question – unknown to whom) in these discussions than the high student interaction codes. These codes were 28.3% of all the student main discussion codes. This is much less than we found in the *dialogic/student*-centered discussions but a slight increase from the *dialogic* discussions.

As I did with the previous two types of discussions I counted the number of SN codes (question/comment is not related to previous student comment). Out of 53 conversations there were fourteen that contained SN codes. However, there were nineteen SN codes meaning that some discussions contained more than one instance. So while there was a smaller percentage of discussions that contained SN codes than in the *dialogic/student-centered* and a comparable percentage to the *dialogic* discussions, these discussions had the first instances of more than one SN code in a single discussion and accounted for 3 times the percent of the main discussion codes than in both the other two types of discussions.

Two of the final codes relating to the role of the student of interest are the SRI code (student responds to question/comment of instructor) and the SQI code (student asks question of instructor). There were 20 occurrences of these codes in 18 of the 53 *student-centered* discussions. The percentage of these codes per discussion is slightly lower than what I found with the *dialogic* discussions.   But in terms of the number per discussion there were two discussions that contained more than one of these codes. In *dialogic* discussions this only occurred once. The most interesting piece of this puzzle is the appearance of SQI codes that were completely absent from the *dialogic* and *dialogic/student-centered* discussions.

In the *dialogic* discussions there were 9 times as many SRS codes as SRI codes and twice as many in the *dialogic/student-centered* discussions. In the *student-centered* discussions there were 101 SRS codes and 16 SRI codes. This is more than six times as many SRS codes as SRI codes.

In terms of the SQI codes and the SQS codes, there were 4 SQI codes and 42 SQS codes. This is 10.5 times more SQS codes than SQI codes. So, although there are now instances of students questioning the instructor, they are still questioning each other 10.5 times more often.

5.3.2 Role of Instructor

In the *student-centered* discussions the instructor main discussion codes only accounted for 22.1 % of the total. This is a decrease from the *dialogic/student-centered* discussions but a definite increase from the 11.1% in the *dialogic* discussions.

I once again split the instructor main discussion codes into the two groups mentioned previously based on the focus of the codes on students as active participants rather than on the instructor. The codes with a focus on the instructor accounted for 40.7% of all instructor main discussion codes. This is down drastically from the *dialogic/student-centered* discussions and down slightly from the 44.2% in the *dialogic* discussions. This leads me to question the previous claim I made that the instructor's focus shifts from that of the students as active participants to the instructor as we move along the discussion continuum towards those discussions with less student-to-student interaction.

These 37 codes included 14 PF codes (the instructor facilitates the discussion by calling on particular students). This is consistent with what I found in the *dialogic* discussions and the *dialogic/student-centered* discussions. It appears that although the percentage of instructor codes that are considered to be focused on the instructor has decreased as we move along the continuum, the percentage of those codes that involve him facilitating the discussion hasn't substantially changed.

In addition there were four PA codes (instructor answers a student's question or addresses a students' comment directly – acts as an authority) which is approximately 1.4% lower than in the *dialogic/student-centered* and 2.7% lower than the percent of these codes in the *dialogic* discussions. There were ten PCM codes (instructor makes a comment). This is almost one-fourth of all the instructor main discussion codes focused on the instructor. This is similar to the *dialogic* discussions, but down from the *dialogic/student-centered* discussions. It would seem that the *dialogic/student-centered* discussions did not follow the patterns I have found in the *dialogic* and the *student-centered* discussions on several occasions.

There were only two PS codes (summarizes or rephrases the presented proof) which did not occur in the *dialogic* discussions but appeared once in the *dialogic/student-centered* discussions. The number of PI codes (instructor interrupts discussion or proof presentation) also increased. There were seven PI codes in the *student-centered* discussions as opposed to two in 31 *dialogic* discussions and one in 9 *dialogic/student-centered* discussions. This is a consistent increase in the number of these codes per discussion as we move away from the high student-to-student interaction discussions along the continuum (although the percentage of instructor main discussion codes deviates from this pattern with the *dialogic/student-centered* discussions).

The other 55 codes (or 59.8% of the instructor codes) were related to behavior consistent with a focus on the students as active participants in the discussion. These included eight PRP codes (redirects a student question/comment to another student or presenter) coming from different discussions. This is an increase in the proportional number of these codes per discussion from the *dialogic* discussions.

The instructor asked the class for questions or comments nineteen times. Unlike the previous two types of discussions, these codes occurred all throughout the discussions – not necessarily at the beginning and the end. Three of these occurred in the middle of a discussion, rather than at the beginning which initiates the discussion or at the end as a way of determining if the discussion has ended. There were eight PQ codes (asks student or presenter a question) and four PFQ codes (asks a follow-up question based on a student response). The percent of PQ codes out of all instructor main discussion codes decreased from the *dialogic* discussions (increased from *dialogic/student-centered*), however the percent of PFQ codes increased from the *dialogic* and *dialogic/student-centered* discussions.

There were four PC codes (asks student to clarify or repeat their comment/question) and six PR codes (rephrases a students comment or question). The proportion of PC codes to total instructor main discussion codes is comparable to that in the *dialogic* discussions. However, the proportion of PR codes to total instructor main discussion codes is much lower in the *student-centered* discussions than it was in the *dialogic* discussions.

The PN code (asks students to give their names) occurred for the first time in the *student-centered* discussions. There were three discussions that involved this code, one of which contained two occurrences. All three discussions occurred within the first week of class.

The final instructor code that is related to a focus on students as active participants in the discussion is the PP code (acts as a peer, contributing equally in the discussion). This code occurred twice, the same as in the *dialogic* discussions but one less than in the *dialogic/student-centered* discussions. In terms of the three types of discussions I considered on the high end of student-to-student interaction I find these numbers low.

5.3.3 Initiating the Discussions

Twenty-two of the 53 *student-centered* discussions were initiated by the instructor. This is approximately 41.5% versus the 32.3% of *dialogic* discussions that were initiated by the instructor. In fourteen of the 31 discussions that were initiated by students the instructor had no codes during the discussion. However, the majority of these came from the shorter discussions in the data set.

5.3.4 End of Discussions

The last piece of data I looked at for the *student-centered* discussions was how the discussions ended. About 60% of the discussions ended in a manner consistent with a focus on the students as active participants in the discussion. This is a slight drop from the *dialogic* and *dialogic/student-centered* discussions. Of the thirty discussions that ended in this way, there were 22 that were coded CS (issue is resolved by the students and Instructor /TA responds to this) or coded CST (issue is resolved by students). This is a slight drop compared to the percent of *dialogic* discussions coded in the same way, however still supports the notion of students resolving the issue when there is high student-to-student interaction in discussions.

The biggest difference was the two CML codes (instructor interrupts a discussion that contains limited student participation to rephrase the issue/comments being made) out of 30 *student-centered* discussions versus the two out of 21 *dialogic* discussions.

Another big difference which is less surprising involved the CMRD codes (instructor interrupts the discussion to redirect the class). These codes generally occurred when a discussion got off topic or a student made a good point that had not been taken-up by the class. This code occurred six times out of 30 *student-centered* discussions as compared to two times out of 21 *dialogic* discussions.

The twenty-three discussions that were ended with a focus on the instructor included six with CMSS codes (instructor interrupts the discussion and begins a discussion with a single student) and one with a CMS code (gives summary of proof – not an interruption). This is a smaller proportion than in the *dialogic* discussions where slightly less than one-third of the discussions that ended in an instructor-focused way ended with a discussion between the instructor and a single student. However, over 50% of the 23 *student-centered* discussions which ended with a focus on the instructor were coded CL (instructor lectures or makes a statement) versus only 30% in the same kind of *dialogic* discussions. There was one discussion with CMR code (instructor interrupts a discussion to rephrase the arguments given).

The results of this analysis of the three high student-to-student interaction discussions are summarized in the two tables below. The first gives the data regarding the role of the students in these discussions (Table 5). The percent of each code out of all the student codes and out of all the main discussion codes is given. Also, for those student codes that were considered high student interaction codes the percent of each out of the total high student interaction codes are given.

Summary of Results from High Student-to-Student Discussions
Student Role Codes (Table 5)

Codes	***Dialogic***	***Dialogic/Student-Centered***	***Student-Centered***
High Student Interaction	69.6% 61.0%	45.5% 31.8%	59.4% 45.4%
SF (make follow-up question/comment after previous question/comment)	20.6% 18.1% 29.7%	12.7% 8.9% 27.9%	13.5% 10.3% 22.8%
SRS/SRP (respond to question/comment of another student or presenter)	35.1% 30.7% 50.4%	26.1% 18.2% 57.4%	31.8% 24.3% 53.4%
SQS/SQP (ask question of another student or presenter)	12.4% 10.9% 17.8%	5.2% 3.6% 11.5%	13.2% 10.1% 22.2%
SC and SQ (student makes comment and student asks question – unknown to whom)	24.5% 21.4%	38.8% 27.1%	28.3% 21.6%
SN (question/comment is not related to previous STUDENT comment)	2.1% 1.8%	2.2% 1.6%	6.0% 4.6%
SRI (respond to question/comment of Instructor)	3.8% 3.4%	13.4% 9.4%	5.0% 3.8%
SQI (ask question of Instructor)	0% 0%	0% 0%	1.3% 1.0%

Proportion of all student main discussion codes (excluding initiating and end of conversation codes)
Proportion of all main discussion codes
Proportion of all HIGH student interaction main discussion codes

In the next table (Table 6) the summary of the analysis of the instructor's role in these discussions is given. The percent of each code out of all instructor main discussion codes, out of all main discussion codes, and out of the corresponding codes based on focus are given. In addition the percent of codes with each of the two focuses out of all the instructor main discussion codes and all main discussion codes are given.

Summary of Results from High Student-to-Student Discussions
Instructor Role Codes (Table 6)

Codes	***Dialogic***	***Dialogic/Student-Centered***	***Student-Centered***
Focus on students as active participants in the discussion			
PC (asks student to clarify or repeat their comment/question)	4.7% 0.5% 8.3%	11.3% 3.1% 21.4%	4.3% 1.0% 7.3%
PR (rephrases a students comment or question)	16.3% 1.8% 29.2%	1.9% 0.5% 3.6%	6.5% 1.4% 10.9%
PFQ (asks a follow-up question based on a student response)	0.0% 0.0% 0.0%	3.8% 1.0% 7.1%	4.3% 1.0% 7.3%
PQ (asks student or presenter a question)	14.0% 1.6% 25.0%	5.2% 4.2% 28.6%	8.7% 2.0% 14.5%
PRP (redirects a student question/ comment to another student or presenter)	7.0% 0.8% 12.5%	3.8% 1.0% 7.1%	8.7% 2.0% 14.5%
PP (acts as a peer, contributing equally in the discussion)	4.7% 0.5% 8.3%	5.7% 1.6% 10.7%	2.2% 0.5% 3.6%
PQC (asks class for questions or comments)	9.3% 1.0% 16.7%	11.3% 3.1% 21.4%	20.7% 4.6% 34.5%
TOTAL:	**55.8%** **6.2%**	**52.8%** **14.6%**	**59.8%** **13.2%**
Focus on instructor			
PS (summarizes or rephrases the presented proof)	0.0% 0.0% 0.0%	1.9% 0.5% 4.0%	2.2% 0.5% 5.4%
PA (answers a student's question or addresses a students' comment directly – acts as an authority)	7.0% 0.8% 15.8%	5.7% 1.6% 12.0%	4.3% 1.0% 10.8%
PI (interrupts discussion or proof presentation)	7.0% 0.8% 15.8%	1.9% 0.5% 4.0%	7.6% 1.7% 18.9%
PCM (makes comment)	11.6% 1.3% 26.3%	22.6% 6.3% 48%	10.9% 2.4% 27.0%
TOTAL:	**44.2%** **4.9%**	**47.2%** **13.0%**	**40.2%** **8.9%**

Proportion of all instructor main discussion codes (excluding initiating and end of conversation codes)
Proportion of all main discussion codes
Proportion of corresponding instructor main discussion codes based on focus

5.4 Student-Centered/Socratic Discussions

The only discussion type that I considered to have a moderate level of student-to-student interaction is the *student-centered/Socratic* discussions which fall in the middle of my discussion continuum in Figure 2. I continued my analysis of the seven discussion types with these discussions.

5.4.1 Role of Students

As before, I looked for codes I considered high student-interaction codes within the main discussion codes and compared the number of occurrences of these codes with all other main discussion codes. I found that 31.1% of the main discussion codes fell into this category of high student interaction. This is a decrease from what I found in *student-centered discussions,* which I expected. I also calculated what percent of these high student interaction codes were the SF code (make follow-up question/comment after previous question/comment). These codes made up 20.0% of the high student interaction codes. In the high student-to-student interaction discussions (*dialogic*, *dialogic/student-centered*, and *student-centered*) these codes ranged from 29.7% to 22.8% of the high student interaction codes. Thus this proportion continues to decrease as we move towards the low student-to-student interaction discussions.

An additional code of interest related to the role of the students in discussions is the SN code (question/comment is not related to previous student comment). In the 18 discussions there were 4 different instances of students making comments unrelated to previous student comments, two of which came from the same discussion. The proportion of these codes to all student main discussion codes is a slight increase from the *dialogic* and *dialogic/student-centered* discussions but a decrease from the *student-centered* discussions.

Other main discussion codes that are relevant to the role of the students are the students making comments (SC), asking a question (of whom is unclear) (SQ), responding to the instructor (SRI), and asking questions of the instructor (SQI) codes. The SC and SQ codes made up 30.2% of all the student main discussion codes. This is an increase from the *dialogic* and *student-centered* discussions (*dialogic/student-centered* discussions had a much higher percentage). As in the *dialogic* and the *dialogic/student-centered* discussions there were no SQI codes in the *student-centered/Socratic* discussions. However, 20.1% of all student main discussion codes were SRI. This is up dramatically from the *student-centered* discussions.

When we compare the number of SRS codes to the number of SRI codes we find that there were only one and one-third times as many SRS codes as SRI codes. These two codes are much closer in count than they were in any of the previous discussions (except the *dialogic/student-centered* discussions which have deviated from the main trend on several occasions).

5.4.2 Role of Instructor

Next I analyzed the main discussion codes representing the role of the instructor in the *student-centered/Socratic* discussions. The proportion of the main discussion codes that are instructor codes increased from the three high student-to-student interaction discussions. In those three types of discussions the percent of all main discussion codes that were instructor codes ranged from 11.1% to 27.6% (the *dialogic/student-centered* discussions were 5.5% higher than the *student-centered*). In the *student-centered/Socratic* discussions this percentage rose to 29.8%. It would appear that as we move further from the highest levels of student-to-student interaction the instructor is more active in the discussions.

Next I took these instructor codes, as I have done previously, and split them into the two categories described previously: focus on students being active participants in the discussion and focus on instructor. The codes that focused on the instructor made up 35.8% of the instructor codes. This percentage is down from the three high student interaction discussions. So although the instructor appears to be more active, the focus is on the instructor less.

There were 24 codes that fell into this category. These included six PF codes (the instructor facilitates the discussion by calling on particular students), two PA codes (the instructor answers a student's question or addresses a students' comment directly – acts as an authority), eleven PCM codes (instructor makes a comment), two PS codes (instructor summarizes or rephrases the presented proof), and three PI codes (instructor interrupts the discussion or proof presentation).

Since the percent of instructor codes that focus on the instructor has decreased I chose to compare the percents of each of these codes out of just the instructor codes that focus on the instructors through all of the four discussion types analyzed thus far. The PCM codes represented 45.8% of all the instructor codes that focused on the instructor in the *student-centered/Socratic* discussions. This is a dramatic increase from both the *dialogic* and the *student-centered* discussions (but a slight decrease from the *dialogic/student-centered* discussions). There was also a significant increase in the percent of these codes that were the PS code. This percentage consistently went up from *dialogic* discussions through *student-centered/Socratic* discussions.

The only other interesting pattern that is continued in the *student-centered/Socratic* instructor codes with a focus on the instructor are the PA codes. The percent of these codes has consistently decreased as a percentage of all the instructor main discussion codes and as a percentage of just the instructor codes with a focus on the instructor.

This leaves us with 43 codes (or 64.2% of the instructor main discussion codes) that related to a focus on students being active participants in the discussion. These codes consisted of five PR codes (instructor rephrases a students comment or question). These PR codes represent approximately 11.6% of all the instructor codes with a focus on the students. This is dramatically lower than in the *dialogic* discussions but higher than both the *dialogic/student-centered* and the *student-centered* discussions.

During the *student-centered/Socratic* discussions the instructor never redirected a student question/comment to another student or the presenter. This is different than in the high student interaction discussions where this occurred in 7.1% to 14.5% of all instructor codes with a focus on the students. There were eleven PQC codes (instructor asks for questions or comments) and all occurred at the very beginning of the discussion. The percentage of PQC codes decreased from the three high student interaction discussions which in contrast showed an increasing trend as the discussions moved towards the low student interaction discussions.

There were 13 PQ codes (instructor asks student or presenter a question). This shows that although the instructor wasn't asking for comments and questions as often, he did ask questions of students or the presenter. The higher percentage of PQ codes than PQC codes indicates that the questions asked by the instructor had a clearer focus in terms of the content or the desired recipient.

There were only four PC codes (instructor asks student to clarify or repeat their comment/question) which is similar in percentage to the *dialogic* and *student-centered* discussions. The percent of PP codes (instructor acts as a peer, contributing equally in the discussion) continued to decrease, although this code has never represented a large percentage in any of the other discussions.

5.4.3 Initiating Discussions

In terms of initiating discussions 8 of the 18 discussions were initiated by students – either the presenter or a non-presenter. In the ten discussions initiated by Dr. Stone, all were initiated with the PQC or PQ codes. Only three of the 10 were initiated with the PQC code. The discussions initiated by the students all began with students making comments, asking questions of each other, or responding to a statement made by another student (usually the presenter). Overall the percentage of discussions that were initiated by Dr. Stone has increased as we move along the continuum. The only exception to this is the *dialogic/student-centered* discussions.

5.4.4 End of Discussions

The codes from the end of the *student-centered/Socratic* discussions were split between a focus on the instructor and a focus on the students as active participants less evenly than the initiating codes were split between instructor and students. There were 12 discussions that ended in a manner consistent with a focus on the students. These included ten CS codes (issue is resolved by the students and Instructor /TA responds to this), and two CST codes (issue is resolved by students). This means that all the discussions that ended with a focus on the students were a result of the students resolving the issues raised in the discussion. This is in contrast to all the other discussions that had a wider variety of discussion endings that focused on the students. The other six discussions ended in a manner consistent with a focus on the instructor. These all were CL codes (instructor lectures or makes a statement).

The results of this analysis of the *student-centered/Socratic* discussions are summarized in the two tables below. The first gives the data regarding the role of the students in these discussions (Table 7). The percent of each code out of all the student codes and out of all the main discussion codes is given. Also, for those student codes that were considered high student interaction codes the percent of each out of the total high student interaction codes are given.

Summary of Results from Moderate Student-to-Student Discussions
Student Role Codes (Table 7)

Codes	***Student-Centered/Socratic***
High Student Interaction	47.0% 31.1%
SF (make follow-up question/comment after previous question/comment)	9.4% 6.2% 20.0%
SRS/SRP (respond to question/comment of another student or presenter)	26.8% 17.8% 57.1%
SQS/SQP (ask question of another student or presenter)	10.1% 6.7% 21.4%
SC and SQ (student makes comment and student asks question – unknown to whom)	30.2% 20.0%
SN (question/comment is not related to previous STUDENT comment)	2.7% 1.8%
SRI (respond to question/comment of Instructor)	20.1% 13.3%
SQI (ask question of Instructor)	0.0% 0.0%

Proportion of all student main discussion codes (excluding initiating and end of conversation codes)
Proportion of all main discussion codes
Proportion of all HIGH student interaction main discussion codes

In the next table (Table 8) the summary of the analysis of the instructor's role in the *student-centered/Socratic* discussions is given. The percent of each code out of all instructor main discussion codes, out of all main discussion codes, and out of the corresponding codes based on focus are given. In addition the percent of codes with each of the two focuses out of all the instructor main discussion codes and all main discussion codes are given.

Summary of Results from Moderate Student-to-Student Discussions
Instructor Role Codes (Table 8)

Codes	***Student-Centered/Socratic***
Focus on students as active participants in discussion	
PC (asks student to clarify or repeat their comment/question)	6.0% 1.8% 9.3%
PR (rephrases a students comment or question)	7.5% 2.2% 11.6%
PFQ (asks a follow-up question based on a student response)	13.4% 4.0% 20.9%
PQ (asks student or presenter a question)	19.4% 5.8% 30.2%
PRP (redirects a student question/ comment to another student or presenter)	0.0% 0.0% 0.0%
PP (acts as a peer, contributing equally in the discussion)	1.5% 0.4% 2.3%
PQC (asks class for questions or comments)	16.4% 4.9% 25.6%
TOTAL:	**64.2%** **19.1%**
Focus on instructor	
PS (summarizes or rephrases the presented proof)	3.0% 0.9% 8.3%
PF (facilitates the discussion by calling on particular students)	9.0% 2.7% 25.0%
PA (answers a student's question or addresses a students' comment directly – acts as an authority)	3.0% 0.9% 8.3%
PI (interrupts discussion or proof presentation)	4.5% 1.3% 12.5%
PCM (makes comment)	16.4% 4.9% 45.8%
TOTAL:	**35.8%** **10.7%**

Proportion of all instructor main discussion codes (excluding initiating and end of conversation codes)
Proportion of all main discussion codes
Proportion of corresponding instructor main discussion codes based on focus

5.5 Socratic Discussions

The *Socratic* discussions are the first of all the low level of student-to-student interaction discussions. Based on the continuum in Figure 2 this means that these discussions have the highest level of student-to-student interaction of all the remaining discussions.

5.5.1 Role of Students

The first step of my analysis was to look for codes I considered high student-interaction codes within the main discussion codes. I then compared the number of occurrences of these codes to the number of all other main discussion codes. I found that only 11.8% of the main discussion codes fell into this category of high student interaction. This is a decrease from what I found in all the other discussions and continues the decreasing trend I have found in three of the previous four discussion types. What is most interesting is how much of a decrease this is. In *student-centered/Socratic* discussions, high student interaction codes made up 31.1% of all the main discussion codes, almost 20 percentage points higher.

I also calculated what percent of these high student interaction codes were the SF code (make follow-up question/comment after previous question/comment). These codes made up 28.0% of the high student interaction codes. In all the other discussions this percentage decreased as we moved along the continuum in the direction of the low student-to-student interaction discussions. However, this is an increase from the *student-centered/Socratic* percentage of SF codes in the high student interaction main discussion codes. Although these made up a similar proportion of the high student interaction codes as they did in the high student interaction discussions, they make up a smaller proportion of all the student discussion codes than they did in the other discussions. This is to be expected due to the decrease in the percentage of high student interaction codes overall.

Other main discussion codes that are relevant to the role of the students are the students making comments (SC) and students asking a question (of whom is unclear) (SQ) codes. The SC and SQ codes made up 33.6% of all the student main discussion codes. This is an increase from three of the four previous discussions. This percentage continues the overall increasing percentage of these codes as we move towards the low student-to-student interaction end of the discussion continuum.

Also of interest are the students asking questions of the instructor (SQI) and the students responding to the instructor (SRI) codes. The SQI codes only made up 2.2% of all the student main discussion codes. In all the previous discussions this percentage has been less than 1.5%. So while this percentage is only 2.2% it is an increase over all the other discussions. The most dramatic change is the increase in the percent of student main discussion codes that are the SRI code, 39.0%. This is up fro 20.1% in the *student-centered/Socratic* discussions and almost 8 times as large a percentage as in the *student-centered* discussions.

When we compare the number of SRS codes to the number of SRI codes we find for the first time that there were more SRI codes than SRS codes. In fact there were 3 times as many SRI codes as SRS codes. It appears that on the low student-to-student interaction end of the continuum a shift has occurred between how often the students respond to each other and how often they respond to the instructor.

A final student main discussion code of interest is the SN code (question/comment is not related to previous student comment). Overall this code has represented a small proportion of all the student main discussion codes. This trend continued in the *Socratic* discussions. Only 2.7% of all the student main discussion codes were SN codes. This is identical to the *student-centered/Socratic* discussions and similar to both the *dialogic* and *dialogic/student-centered* discussions. However, this percentage is half the percentage of SN codes in the *student-centered* discussions.

5.5.2 Role of Instructor

Next I analyzed the main discussion codes representing the role of the instructor in the *Socratic* discussions. The proportion of the main discussion codes that are instructor codes continued to increase from the previous four discussions. The percentage of main discussion codes that were instructor codes was 41.5%. This is up from 29.8%, the percentage of instructor codes in the *student-centered/Socratic* discussions.

The codes that focused on the instructor made up 44.3% of the instructor codes. This is up from the *student-centered/Socratic* discussions but is comparable to the percentages in the three high student interaction discussions. There were 78 codes that fell into this category. These included thirteen PF codes (the instructor facilitates the discussion by calling on particular students), twenty-one PA codes (the instructor answers a student's question or addresses a students' comment directly – acts as an authority), thirty-seven PCM codes (instructor makes a comment), five PS codes (instructor summarizes or rephrases the presented proof), and two PI codes (instructor interrupts the discussion or proof presentation).

Once again, the PCM codes represented close to 50% of all the instructor codes that focused on the instructor in the *Socratic* discussions. This is similar to what occurred in the *student-centered/Socratic* discussions but continues to be a dramatic increase from both the *dialogic* and the *student-centered* discussions (but a slight decrease from the *dialogic/student-centered* discussions). There was also a significant increase in the percent of these codes that were the PS code. This percentage consistently went up from *dialogic* discussions through *student-centered/Socratic* discussions.

The percentage of PI codes (instructor interrupts discussion or proof presentation) has dropped from 4.5% of all instructor codes (in the *student-centered/Socratic* discussions) to 1.1% of all instructor codes. There is also a drop in the percentage of these codes with respect to the instructor codes that focus on the instructor. This percentage is also down from all three of the high student-to-student interaction discussions.

Also of interest is the pattern described previously in regards to the PA code. In the four prior discussions I noticed a decreasing pattern in the percent of PA codes out of the instructor codes as well as out of just the instructor codes with a focus on the instructor. The *Socratic* discussions completely contradict this pattern with a dramatic increase in these percentages. The percentage of instructor codes that were PA is 11.9%. In addition these codes represented more than one-fourth of all the instructor codes with a focus on the instructor.

This leaves us with 98 codes (or 44.3% of the instructor main discussion codes) that related to a focus on students being active participants in the discussion. These codes included fourteen PR codes (instructor rephrases a students comment or question). This is approximately 14.3% of all the instructor codes with a focus on the students. This is only a slight increase than in the *student-centered/Socratic* discussions. There also was a slight increase in the percent of PRP codes (instructor redirects a student question/comment to another student or the presenter) from 0% of all instructor codes to 2.3% of all instructor codes and 4.1% of all instructor codes with a focus on the students. Although this is a slight increase from the *student-centered/Socratic* discussions it is lower than in the three high student-to-student discussions.

There were nine PQC codes (instructor asks for questions or comments) and all occurred at the very beginning or end of the discussion. The percentage of PQC codes decreased dramatically from the *student-centered/Socratic* discussions (which consequently is much lower than all the high student-to-student discussions). These codes were 9.2% of the instructor codes that focus on the students as compared to the 25.6% from the *student-centered/Socratic* discussions. There was also a decrease in the percentage of PQ codes (instructor asks student or presenter a question). These dropped from 30.2% of all instructor codes with a focus on the students to 28.6% of these same codes. In the high student-to-student discussions this percentage ranged from 14.5% to 28.6%, so this percentage is comparable to these discussions.

There were fourteen PC codes (instructor asks student to clarify or repeat their comment/question) which is similar in percentage to the *dialogic* and *student-centered* discussions but lower than the other two discussions (*dialogic/student-centered* and *student-centered/Socratic*).

One of the most significant changes was in the proportion of PFQ codes among the instructor codes with a focus on students. There were 32 PFQ codes (instructor asks a follow-up question based on a student response). These represented 32.7% of all the instructor codes with a focus on the students and 18.2% of all instructor codes in the *Socratic* discussions. This is up from 20.9% of all instructor codes with a focus on the students in the *student-centered/Socratic* discussions. However, this may be obvious based on how the discussions were coded. *Socratic* discussions involved limited student interaction, so the majority of questions asked, especially those based on a student's response, were asked by the instructor.

5.5.3 Initiating Discussions

In terms of initiating discussions 12 of the 33 discussions were initiated by students. In the twenty-one discussions initiated by Dr. Stone or the TA, fourteen began with him asking for questions or comments indicated by the PQC or PQ codes. In the other 7, four of these were initiated with the PCM code (instructor makes a comment), two were with the PS code (instructor summarizes the presented proof), and one with the PC code (instructor asks student to clarify or repeat their comment/question). Unlike the *student-centered/Socratic* discussions, there was a higher percentage of discussions that were initiated by the instructor and among these there were more discussions initiated in a way that focused on the instructor rather than the students. The eleven discussions initiated by the students all began with students making comments (3 discussions) or asking questions (not necessarily of each other).

5.5.4 End of Discussions

The codes from the end of the *Socratic* discussions were split between a focus on the instructor and a focus on the students as active participants more evenly than the initiating codes were split between instructor and students. There were 14 discussions that ended in a manner consistent with a focus on the students. These included nine CS codes (issue is resolved by the students and Instructor /TA responds to this), and three CST codes (issue is resolved by students. This means that 86% of the discussions that ended with a focus on the students were a result of the students resolving the issues raised in the discussion. There were also two discussions that ended with the CMRD code (Instructor interrupts the discussion to redirect the class). However, the high percentage of discussions resolved by students (at least among those which ended with a focus on students) is consistent with what I have found in the other discussion types.

The other nineteen discussions ended in a manner consistent with a focus on the instructor. There were two CMR codes (instructor interrupts a discussion to rephrase the arguments given), two CMS codes (instructor gives summary of proof – not an interruption), eleven CL codes (lecture or make statement), and four CMSS codes (instructor interrupts the discussion and begins a discussion with a single student). This is a larger variety of these codes than in any other discussion type and is in greatest contrast to what occurred in the *student-centered/Socratic* discussions where all the ending codes that had a focus on the instructor were CL codes.

The final two low student-to-student interaction discussion types had very few examples to draw data from. This shows that most of the discussions in this course that involved two or more students with at least three utterances tended to involve moderate to high student-to-student interaction.

5.6 Socratic/Univocal Discussions

There were only 4 discussions that were coded as *Socratic/univocal* discussions.

5.6.1 Role of Students

Although there was such a small sample to analyze, I found that even fewer of the student main discussion codes involved high student-to-student interaction, 15.4%. However, the percent of these codes that were SF codes (student makes a follow-up question/comment after previous question/comment) was 25.0% which is comparable to all the other discussions. So even though the students are interacting less in these discussions they are asking follow-up questions or making follow-up comments approximately the same proportion of the time that they are involved in these high student interaction behaviors.

The percent of SC and SQ codes continued to rise in the *Socratic/univocal* discussions. These codes accounted for 42.3% of all the student main discussion codes. This percentage is up from 33.6% in the *Socratic* discussions. Generally these codes represent little to no interaction between the students, as opposed to the SRS and SQS codes, and thus this increase is not surprising.

As I did with the previous two types of discussions I counted the number of SN codes (question/comment is not related to previous student comment). Out of the four discussions there was only one SN code. However, this represented 3.8% of all student main discussion codes. In four of the five other discussions previously analyzed this percentage was less than 3%. This is not a large difference, but it is an increase in percentage.

I also determined the percent of all student main discussion codes that were either the SQI code (student asks question of instructor) or the SRI code (student responds to question/comment of instructor). I found that the percent of SQI codes rose slightly. In the *Socratic* discussions this percent was 2.2% and was either 0% or less than 1.5% in all other discussions. While these percentages are low, there is clearly a difference between no SQI codes and 3.8% (from the *Socratic/univocal* discussions) of all student main discussion codes consisting of SQI codes. The SRI codes represented 34.6% of all student main discussion codes. This is down slightly from the *Socratic* discussions; however is an increase of more than 10% from all the other discussions. Although there seem to be some deviations from the pattern, generally these two codes are becoming more prevalent in the discussions on the low student-to-student interaction end of the discussion continuum in Figure 2.

I found in the *Socratic* discussions that there were three times as many SRI codes as SRS codes, whereas in the high student interaction discussions there were more SRS codes than SRI codes. This trend continued with the *Socratic/univocal* discussions. In the four discussions there were nine SRI codes and only one SRS code. This means that there were nine times as many SRI codes as SRS codes. Recall that in the *dialogic* discussions, on the other end of the continuum, there were nine times as many SRS codes as SRI codes. Also of interest is the fact that the percent of all high student interaction main discussion codes that were SRS codes dropped dramatically in these discussions. In most of the other discussions they represented approximately 50% of all the high student interaction codes, yet in the *Socratic/univocal* discussion this percentage dropped to 25%. While this is dramatic, I also recognize that there were only 4 of these discussions.

5.6.2 Role of Instructor

Next I analyzed the main discussion codes representing the role of the instructor in the *Socratic* discussions. The proportion of the main discussion codes that are instructor codes continued to increase from the previous four discussions. The percentage of main discussion codes that were instructor codes was 49%. This is up from 41.5%, the percentage of instructor codes in the *Socratic* discussions.

The codes that focused on the instructor made up 52% of the instructor codes. This is up from all the other discussions. There were only 25 total instructor codes, and 13 fell into this category. These included one PF code (the instructor facilitates the discussion by calling on particular students), five PA codes (the instructor answers a student's question or addresses a students' comment directly – acts as an authority), six PCM codes (instructor makes a comment), and one PS code (instructor summarizes or rephrases the presented proof).

As in the *student-centered/Socratic* and the *Socratic* discussions, the PCM codes represented close to 50% of all the instructor codes that focused on the instructor in the *Socratic/univocal* discussions. There was also a significant increase in the percent of these codes that were the PA code. This code represented less than 20% of all the instructor codes that focused on the instructor in the three high student-to-student interaction discussions and was less than 30% in the *student-centered/Socratic* and *Socratic* discussions. In the *Socratic/univocal* discussions this percentage went up to 38.5%.

For the first time in any of the discussions the PI code was not present. In all the other discussions this code represented 2.6% to 18.9% of all instructor codes that focus on the instructor. This may be because in *Socratic/univocal* discussions, which occur on the end of the continuum with low student-to-student interaction, there are no situations that the instructor feels he needs to interrupt.

This leaves us with 12 codes (or 48% of the instructor main discussion codes) that related to a focus on students being active participants in the discussion. These codes consisted of three PR codes (instructor rephrases a students comment or question). This is approximately 25% of all the instructor codes with a focus on the students. This is an increase from the *Socratic* discussions. There also was an increase in the percent of PRP codes (instructor redirects a student question/comment to another student or the presenter) from 2.3% of all instructor codes to 4.0% of all instructor codes and 8.3% of all instructor codes with a focus on the students. Although this percentage is increasing as we move among the low student interaction discussions it is still a lower percent than in the three high student-to-student discussions.

There was only one PQC code (instructor asks for questions or comments) and one PQ code (instructor asks student or presenter a question). However, there were five PFQ questions (instructor asks a follow-up question based on a student response). This is an increase from the 32.7% of all the instructor codes with a focus on the students in the *Socratic* discussions to 41.7%. So although the instructor is not asking general questions or for questions or comments from the students, he is asking follow-up questions based on the comments and questions the students make.

5.6.3 Initiating Discussions

In terms of initiating discussions 3 of the 4 discussions were initiated by Dr. Stone. Once again we see that a majority of the *Socratic/univocal* discussions were initiated by the instructor as in the *Socratic* discussions. These all began with a different code. Two began with either the PQ or PQC code and one began with the PCM code. This proportion of PQ/PQC codes to PCM codes is fairly consistent with what we found in the *Socratic* discussions. We also see that the majority of these discussions were started with codes that were consistent with a focus on the students as active participants in the discussion. The only discussion initiated by a student came from students asking a question as the instructor was making a comment, so was very close to being initiated by the instructor.

5.6.4 End of Discussions

The *Socratic/univocal* discussions mainly ended with a focus on the instructor. One of these discussions ended because time ran out. Two of the other three ended with the CL code (instructor lectures or makes a comment). The only discussion that ended with a focus on the students was with the CMRD code (Instructor interrupts the discussion to redirect the class). This is the first time that no discussions ended with the CS or CST codes which represent the student resolving the issue of the discussion.

5.7 Univocal Discussions

As with the *Socratic/univocal* discussions, there were not very many *univocal* discussions. There were 12 discussions and only 63 total main discussion codes. Of these, slightly more than half were student codes (32).

5.7.1 Role of Students

I found that only 9.4% of all the student codes were high student interaction codes (students responding to students, questioning students, directing the discussion, and making follow-up comments or questions). This is lower than in any of the other discussions. As we moved towards the low student-to-student interaction end of the discussion continuum this percent has dropped fairly consistently.[13]

We found in all the previous discussions the SF code (student makes a follow-up question/comment after previous comment) represented approximately one-fourth of these high student interaction codes, even when the proportion of high student interaction codes decreased. This pattern continued with the *univocal* discussion, and in fact this percentage was at its highest: 33.3% of all high student interaction codes were the SF codes.

In addition the percent of SC and SQ codes (student makes a comment and student asks a question – unknown to whom) out of all the student main discussion codes was at its highest in the *univocal* discussions. This percent rose from 42.3% to 46.9%. In all the other discussions this percent was less than 40%. The SN codes represented 9.4%

13 The only discussions that deviated from this pattern were the *dialogic/student-centered* discussions.

of all student main discussion codes and this also is the highest percentage out of all the discussions.

The appearance of SRI codes did drop in the *univocal* discussions from the *Socratic* and the *Socratic/univocal* discussions. In the *univocal* discussions these codes represented 28.1% of all the student main discussion codes. While this is significantly higher than in the high student-to-student interaction discussions, it is the lowest of the low student-to-student interaction discussions. However, the *univocal* discussions are described to have no give and take, meaning there is very limited, if any, responding to anyone. The percent of student main discussion codes that were the SRS codes was 6.3%. This is up slightly from the *Socratic/univocal* discussions, but is much lower than in the high student-to-student discussions. In these twelve discussions there were nine SRI codes and 2 SRS codes. This is four and a half times more SRI codes than SRS codes. While this is a smaller difference than in the *Socratic/univocal* discussions, it is lower than all the other discussions (particularly the high student-to-student discussions where there were more SRS codes than SRI codes).

A final student code of interest is the SQS code (student asks a question of another student/or presenter). There were no SQS codes in the *univocal* discussions. This is the first time this has happened – in all other discussions there was at least one SQS code.

5.7.2 Role of Instructor

Next I analyzed the main discussion codes representing the role of the instructor in the *univocal* discussions. The proportion of the main discussion codes that are instructor codes decreased from the previous two discussions. The percentage of main discussion codes that were instructor codes was 39.6%. This is down from 49%, the percentage of instructor codes in the *Socratic/univocal* discussions and 41.5% in the *Socratic* discussions.

The codes that focused on the instructor made up 40% of all the instructor main discussion codes. This is the lowest of all the low student-to-student discussions (as well as the three high student interaction discussions) and is only higher than the moderate student-to-student interaction discussions (*student-centered/Socratic*). There were only 30 total instructor codes, and 12 fell into this category. These included two PF codes (the instructor facilitates the discussion by calling on particular students), three PA codes (the instructor answers a student's question or addresses a students' comment directly – acts as an authority), six PCM codes (instructor makes a comment), and one PS code (instructor summarizes or rephrases the presented proof).

The PCM codes represented 50% of all the instructor codes that focused on the instructor. In the other two low student-to-student interaction discussions this percent was a little lower, but still close to 50%. The percent of PA codes out of the instructor codes with a focus on the instructor dropped from the *Socratic/univocal* discussions, however it is still close to the percentage for the *Socratic* discussions and is much lower than all the other discussions. As in the *Socratic/univocal* discussions the PI code was not present.

This leaves us with 18 codes (or 60% of the instructor main discussion codes) that related to a focus on students being active participants in the discussion. These codes consisted of three PR codes (instructor rephrases a students comment or question). This is approximately 16.7% of all the instructor codes with a focus on the students and 10% of all instructor main discussion codes. This is an increase from the *Socratic* discussions, although it is a decrease from the *Socratic/univocal* discussions. There were two PRP codes (instructor redirects a student question/comment to another student or the presenter) which are 6.7% of all the instructor codes for the *univocal* discussions. This is up slightly from the *Socratic/univocal* and *Socratic* discussions. So although the percent of PRP codes tends to be lower in the low student-to-student interaction discussions as opposed to the high student-to-student interaction discussions, there is an increasing trend among the low interaction discussions as we move from high to low student interaction.

There were four PQC codes (instructor asks for questions or comments) and four PQ codes (instructor asks student or presenter a question). These codes, together, represented 44.4% of all instructor codes with a focus on the students and 26.6% of all instructor codes. This is up from the other low interaction discussions. In addition there were two PFQ codes (instructor asks a follow-up question based on a student response). This is a decrease from the 32.7% of all the instructor codes with a focus on the students in the *Socratic* discussions and the 41.7% percent in the *Socratic/univocal* discussions. In addition to a drop in the proportion of instructor codes involving follow-up questions, there were no PC codes (instructor asks student to clarify or repeat their comment/question).

5.7.3 Initiating Discussions

In terms of initiating discussions 10 of the 12 discussions were initiated by Dr. Stone. This is a much larger proportion of discussions initiated by the instructor than in any other discussion type. Six of the twelve began with either the PQ or PQC code. Only two began with the PCM code, however one began with the PS code (instructor summarizes or rephrases the presented proof). The last discussion that was initiated by the instructor began with the PF code (the instructor facilitates the discussion by calling on students). This means that half of these discussions that were initiated by the instructor were initiated in a manner consistent with a focus on the students as active participants in the discussion and the other half with a focus on the instructor. The two discussions that were initiated by a student involved students asking a question.

5.7.4 End of Discussions

The codes from the end of the *univocal* discussions were evenly split between those with a focus on the instructor and those with a focus on the students as active participants in the discussion. Of those discussions ended in a manner consistent with a focus on the instructor, four ended with the CL code (instructor lectures or makes a comment), one with the CMR code (instructor interrupts a discussion to rephrase the arguments given), and one with the CMSS code (instructor interrupts the discussion and begins a discussion with a single student).

Of the six discussions that ended with a focus on the students, only two were CS codes (issue is resolved by the students and instructor/TA acknowledges this). There were three that ended with the CMRD code (Instructor interrupts the discussion to redirect the class) and one that ended with the CML code (instructor interrupts a discussion that contains limited student participation to rephrase the issue/comments being made). This final code seems to fit with the *univocal* discussions since these are defined to have limited student-to-student interaction.

The results of this analysis of the three low student-to-student interaction discussions are summarized in the two tables below. The first gives the data regarding the role of the students in these discussions (Table 9).

Summary of Results from Low Student-to-Student Discussions
Student Role Codes (Table 9)

Codes	*Socratic*	*Socratic/Univocal*	*Univocal*
High Student Interaction	22.4% 11.8%	15.4% 7.8%	9.4% 4.8%
SF (make follow-up question/comment after previous question/comment)	6.3% 3.3% 28.0%	3.8% 2.0% 25.0%	3.1% 1.6% 33.3%
SRS/SRP (respond to question/comment of another student or presenter)	13.0% 6.8% 58.0%	3.8% 2.0% 25.0%	6.3% 3.2% 66.7%
SQS/SQP (ask question of another student or presenter)	3.1% 1.6% 14.0%	7.7% 4.0% 50.0%	0.0% 0.0% 0.0%
SC and SQ (student makes comment and student asks question – unknown to whom)	33.6% 17.6%	42.3% 21.6%	46.9% 23.8%
SN (question/comment is not related to previous STUDENT comment)	2.7% 1.4%	3.8% 2.0%	9.4% 4.8%
SRI (respond to question/comment of Instructor)	39.0% 20.5%	34.6% 17.6%	28.1% 14.3%
SQI (ask question of Instructor)	2.2% 1.2%	3.8% 2.0%	6.3% 3.2%

Proportion of all student main discussion codes (excluding initiating and end of conversation codes)
Proportion of all main discussion codes
Proportion of all HIGH student interaction main discussion codes

In the next table (Table 10) the summary of the analysis of the instructor's role in these discussions is given.

Summary of Results from Low Student-to-Student Discussions
Instructor Role Codes (Table 10)

Codes	*Socratic*	*Socratic/Univocal*	*Univocal*
Focus on students as active participants in the discussion			
PC (asks student to clarify or repeat their comment/question)	4.0% 1.6% 7.1%	0.0% 0.0% 0.0%	0.0% 0.0% 0.0%
PR (rephrases a students comment or question)	7.8% 3.3% 14.3%	12.0% 5.9% 25.0%	10.0% 4.8% 16.7%
PFQ (asks a follow-up question based on a student response)	18.2% 7.5% 32.7%	20.0% 9.8% 41.7%	6.7% 3.2% 11.1%
PQ (asks student or presenter a question)	15.9% 6.6% 28.6%	4.0% 2.0% 8.3%	13.3% 6.3% 22.2%
PRP (redirects a student question/ comment to another student or presenter)	2.3% 0.9% 4.1%	4.0% 2.0% 8.3%	6.7% 3.2% 11.1%
PP (acts as a peer, contributing equally in the discussion)	1.7% 0.7% 3.1%	4.0% 2.0% 8.3%	0.0% 0.0% 0.0%
PQC (asks class for questions or comments)	5.1% 2.1% 9.2%	4.0% 2.0% 8.3%	13.3% 6.3% 22.2%
TOTAL:	**55.7%** **23.1%**	**48.0%** **23.5%**	**60.0%** **20.6%**
Focus on instructor			
PS (summarizes or rephrases the presented proof)	2.8% 1.2% 6.4%	4.0% 2.0% 7.7%	3.3% 1.6% 8.3%
PF (facilitates the discussion by calling on particular students)	7.4% 3.1% 16.7%	4.0% 2.0% 7.7%	6.7% 3.2% 16.7%
PA (answers a student's question or addresses a students' comment directly – acts as an authority)	11.9% 4.9% 26.9%	20.0% 9.8% 38.5%	10.0% 4.8% 25.0%
PI (interrupts discussion or proof presentation)	1.1% 0.5% 2.6%	0.0% 0.0% 0.0%	0.0% 0.0% 0.0%
PCM (makes comment)	21.0% 8.7% 47.4%	24.0% 11.8% 46.2%	20.0% 9.5% 50.0%
TOTAL:	**44.3%** **18.4%**	**52.0%** **25.5%**	**40.0%** **19.0%**

Proportion of all instructor main discussion codes (excluding initiating and end of conversation codes)
Proportion of all main discussion codes
Proportion of corresponding instructor main discussion codes based on focus

In terms of initiating and ending the discussions, there are some interesting trends as we move along the continuum. I have summarized this data in the tables below.

Initiating Discussions

Discussion	Student - Initiated	Instructor-Initiated
Dialogic	**67.7%**	**33.3%**
Dialogic/Student-Centered	33.3%	67.7%
Student-Centered	**58.5%**	**41.5%**
Student-Centered/Socratic	44.4%	55.6%
Socratic	**36.4%**	**63.6%**
Socratic/Univocal	25%	75%
Univocal	**16.7%**	**83.3%**

Table 11

End of Discussions

Discussion	Focus on Students as active participants	Focus on Instructor
Dialogic	**67.7%**	**32.3%**
Dialogic/Student-Centered	66.7%	33.3%
Student-Centered	**56.6%**	**43.4%**
Student-Centered/Socratic	66.7%	33.3%
Socratic	**42.4%**	**57.6%**
Socratic/Univocal	25%	75%
Univocal	**50%**	**50%**

Table 12

We can see in Table 11 that among the four main discussion types there is a decreasing trend in students initiating the discussions and an increasing trend in the instructor initiating discussions. In terms of the end of the discussions (Table 12), there is also a decreasing trend in the focus of these on students as active participants with the exception of the *univocal* discussions.

6. Discussion

After my analysis was complete I was able to add more detailed descriptions of the roles and behaviors of both the students and the instructor that took place in each of the four main discussion types from my original discussion continuum. The revised continuum is given in Figure 5.

REVISED: Continuum of discussions involving two or more students

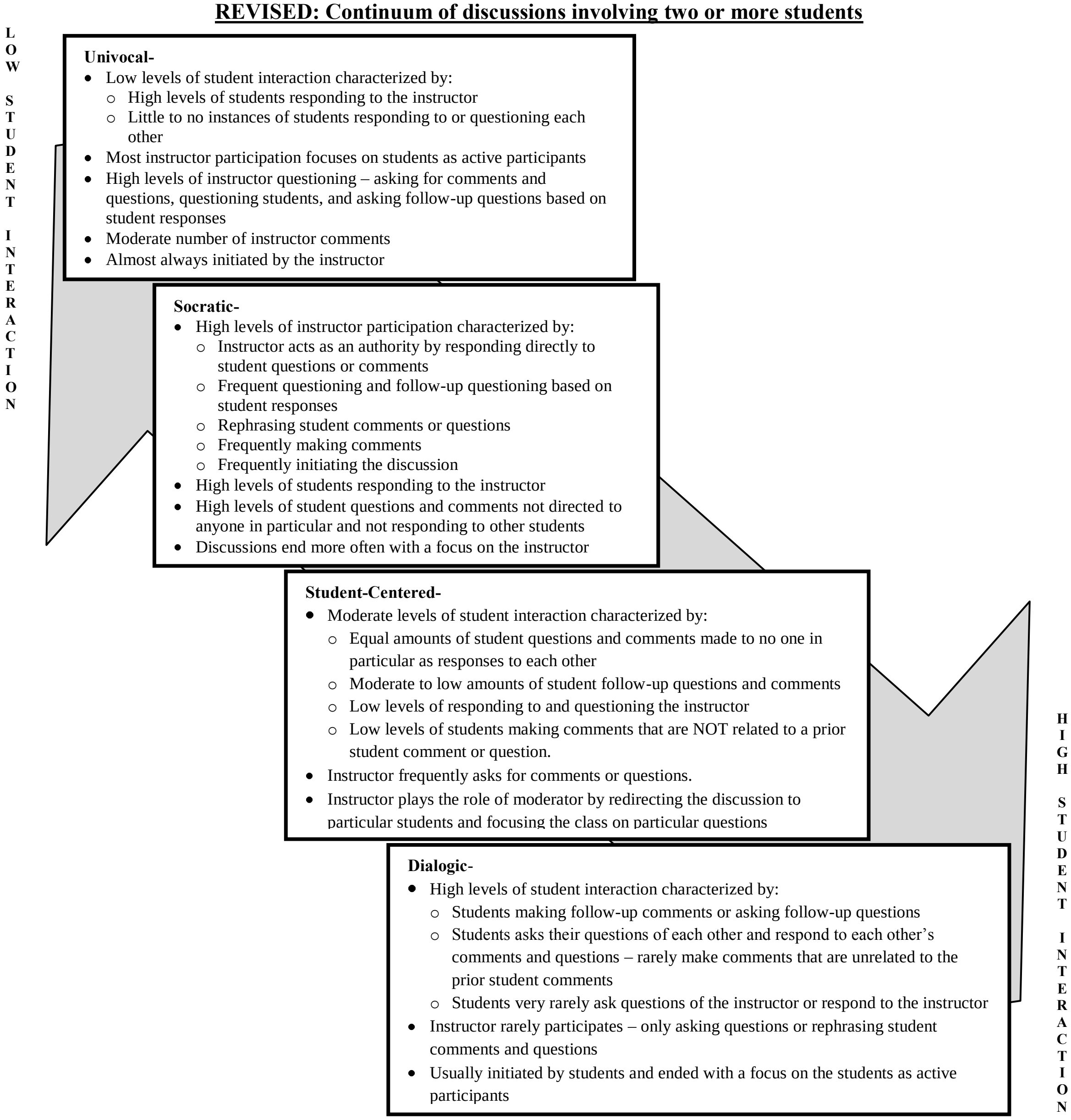

Figure 5

During the course of the analysis of the seven different types of discussions I observed several trends that occurred as I moved along the discussion continuum from the high student-to-student interaction discussions to the low student interaction discussions.

6.1 Overall trends between discussions

The trends that I observed were not observed in all of the seven discussion types. However, generally the four main discussion types in the continuum did follow these patterns. Some of these trends were expected based on the initial design of the continuum and some were not.

6.1.1 Overall trends between discussions – student behaviors

During the analysis of my data I observed several trends that supported my expectations based on the initial continuum (see Figure 2). I suspected that the proportion of student behaviors that supported a high level of interaction would be highest in the *dialogic* discussions and decrease as we moved along the continuum towards the *univocal* discussions and I found this to be the case in this data. The only exception to this was the *dialogic/student-centered* discussions; however many of these "in-between" discussion types had very few samples to analyze.

Another trend that I found in the data, that supported my expectations, was the increasing proportion of SC and SQ codes which indicated an increasing amount of instances where the students made comments that were not directly in response to another student's statements and asked questions that were not directed to anyone, in particular not another student. I expected as the level of interaction declined, these behaviors would increase, as was the case.

The proportion of time that the students responded to the instructor spiked during the *Socratic* discussions. There were low levels in the three high student-to-student interaction discussions, but these generally increased as we moved towards the *Socratic* discussions. I am not surprised that these behaviors spiked during the *Socratic* discussions because these were defined to be the discussions that involved some give and take, however mainly between students and the instructor. This implies that the students would be responding to the instructor much more than to each other, which is what I found in the data. In fact, the students responded to the instructor three times more often than responding to each other during the *Socratic* discussions.

The final trend involving student behaviors that was not surprising was the increasing proportion of instances when the students asked the instructor questions. There were limited, if any, times when students asked the instructor questions in the four discussions with the highest levels of student interaction. Clearly if students are interacting with each other at a high level, there must be less interaction with the instructor, implying they are asking the instructor fewer questions. When you look at just the three low student-to-student interaction discussions, students asked questions of the instructor more often as we move along the continuum from the *Socratic* discussions to the *univocal* discussions.

6.1.2 Overall trends between discussions – instructor behaviors

There were fewer overall trends within the instructor behaviors than there were in the student behaviors. It appears, from the data, that one of the biggest trends was not necessarily the specific instructor behaviors but rather the quantity of them. In general, the proportion of coded behaviors that were instructor behaviors increased as we went from the high student interaction end of the continuum to the low end of the continuum. There were two exceptions to this. The first is the *dialogic/student-centered* discussions; however these seemed to cause problems with a lot of the trends I found. The second exception came from the *univocal* discussions. This might seem surprising since these discussions involve "one voice". However, this was never restricted to the instructor's voice, therefore the low number of instructor behavior codes indicates that the students were doing most of the talking, however they were not talking with each other.

The only real trends that I found in the instructor codes involved the instructor asking follow-up questions, summarizing or rephrasing the presented proof, and interrupting the discussion. Overall there seemed to be an increasing trend in terms of the instructor asking follow-up questions, even though there was a spike in the proportion of times this occurred in the *student-centered/Socratic* discussions.

In terms of the instructor summarizing the presented proof, this tended to increase during the low student-to-student interaction discussions versus the high interaction discussions. This rarely happened during the high interaction discussions, and in fact never happened during the *dialogic* discussions, and only increased slightly during the low interaction discussions. When we think about the context of the high student-to-student interaction discussions, it would seem that the instructor summarized these proofs less (if at all) because the students were doing this or they were able to immediately focus on an issue in the proof and begin a discussion about that issue.

The final instructor code that seemed to follow a trend was the instructor interrupting the discussion. This happened more often in the high student-to-student interaction discussions. This rarely happened, if at all, in the low student interaction discussions. If we consider the brief descriptions of these discussions in the continuum we see that there is little interaction between students and most interaction that does occur is between the instructor and students. If there is limited interaction between the students, the instructor does not appear to need to interrupt the discussion when he chooses to talk. On the other end of the continuum there are high levels of student interaction, so it may be that the instructor has to interrupt a discussion between students in order to make a statement or ask a question. In addition, I found that some of these discussions seemed to have low numbers of participants, even though the average number of participants was not on the low end. It may have appeared that there were fewer participants because the students in the discussion were a small subset of the class and the interaction between them was quite high making it seem as though more students were being left out of the discussion. The instructor may have sensed this and interrupted in order to include more students in the discussion.

6.1.3 Overall trends – content

In addition to the trends that I observed related to instructor behaviors, there were some trends related to the difficulty of the content being discussed. The results of my analysis of the content for the four main discussions were given in the previous chapter. One of the most interesting observations was the high percentage of the *dialogic* discussions that involved difficult content. However, it is interesting to note that the discussions with the lowest proportion of the low content discussions were the *Socratic* discussions, which involve high levels of instructor participation and leadership. It appears that although it is clear that the students are capable of talking about difficult content in a *dialogic* manner, there is still a sense of a need for instructor guidance, either on the part of the instructor, the students, or both, almost as often when talking about difficult content.

On the other end of the spectrum are the *univocal* discussions which have the highest proportion of medium difficulty and low difficulty content. It would appear that the content that is considered easier was more often disposed of in a brief *univocal* discussion, but rarely was the difficult content dealt with in such a brief manner.

6.2 Observations of individual behavior codes

In addition to the patterns I found as I moved along the discussion continuum there were some observations I made of particular codes overall that were somewhat surprising. Of course this may be due to my knowledge of the video from the entire semester and this impacting what I might expect in the subset of data I have analyzed here.

6.2.1 – Interesting observations of individual behavior codes - instructor

Several of the instructor codes occurred at much lower levels than I expected. I was surprised by the low levels of PC codes, where the instructor asks the student to clarify or repeat their comment or question. This was especially surprising when I related it to the proportion of PR codes, where the instructor rephrases a student's comment or question. I expected in this type of inquiry-oriented course that there would be more emphasis on students clarifying their ideas rather than the instructor doing this.

In addition I found there to be a higher proportion of PA codes (instructor answers a student's question or addresses a student's comment – acts as an authority) than I expected. While these never accounted for more than 20% of the codes, and in the high student interaction discussions – 7%, this seems to mismatch the goal of the course to place the authority and ownership of ideas in the hands of the students. Of course, all of these instances may have been necessary as the instructor represented the mathematical community and there are some concepts, especially notation, that students would not be able to develop on their own.

In connection with my surprise at the large proportion of PA codes was a surprise at the low level of PP codes (the instructor acts as a peer, contributing equally in the discussion). These codes were never more than 6% of all the instructor codes. It may be that as the students are interacting well with each other the instructor did not see the need to insert himself into the discussion. In addition, this may tie in with the higher proportion of PA codes and the idea of the instructor as an authority and whether this idea of authority can ever truly be completely transferred to the students. It seems that in this class, although the students were becoming better at being their own authorities, they still saw the instructor as an authority rather than as a peer contributing equally in the discussions.

6.2.2 – Interesting observations of individual behavior codes - students

There weren't as many student behavior codes that were as surprising. One of the most interesting was the SRS code (students responding to students). When I looked at the proportion of these codes out of all student discussion codes this value dropped as I moved along the continuum from the high interaction end to the low interaction end. However, when I just look at what percentage of the high interaction codes these were I find that this percentage remained fairly consistent along the continuum. In fact, when I look at all three of the high student interaction codes, SF, SRS, and SQS, the proportion of these among just the high student interaction codes remained very consistent throughout all the discussions. So although the proportion of time that students were participating in ways that involved high student interaction decreased, the way this time was distributed among those three codes stayed the same.

Another interesting observation I made had to do with the SN code (student question/comment is not related to previous student comment). I expected that as I moved towards the low interaction end of the continuum this number would rise significantly. However, the proportion of these codes was always less than 10% and usually less than 5% of all student codes. It seems that even though students tended not to ask questions directly of each other or to respond directly to each other on the low interaction end of the continuum, they still heard each other and did not attempt to change the course of the discussion away from the issue their classmate (or instructor) had raised.

The last observation I had came from comparing the proportion of SQS codes to SQ/SC codes. It appears that students may not have felt as comfortable asking questions directly of each other. A lot of the SC codes came from students "asking a question" in the form of a comment. This may have been done in order to avoid, what they might consider, looking foolish if their question was easily answered. It would be interesting in future research to explore the SC code in more detail.

6.3 Student-to-student discussion descriptions

In order to better describe the four main discussion types I have included transcribed clips that reflect what I found in the Analysis chapter for each of these discussions.

<u>6.3.1 Dialogic discussions</u>

The following clip is from November and is an example of a *dialogic* discussion. In general, this clip shows how students respond to each other, the high levels of follow-up, as well as the limited participation of the instructor except in the beginning and the end of the discussion. In particular, this discussion ended with the instructor interrupting the discussion in order to rephrase the issue being discussed and attempt to resolve the issue that has not been resolved by all students involved (although it appears to be for some of the students). As before, the instructor is represented by *M*.

M: Mi?

Mi: Is, if you're proving by contradiction, don't you have to prove that it cannot be possible for every case in order. Like cause if you go up you're proving, can you move it up R? You're proving that it cannot, that b cannot be any integer, but it seems in this proof he is only proving (*M* has sat down in the back of the room) for integers in the form of b factorial. Like can b be three which cannot be expressed in terms of a factorial?

M: R?

R: (Pause). I don't really understand what you're saying.

Mi: Like if you have e equals a over 3, can you express 3 in terms of a factorial in order to use your proof?

L: No, you would multiply each side by 2 factorial and you'd have three factorial e. You're not –

Mi: But you're not proving it for a over 3 you're proving it for a over 3 factorial.

L: No.

A: E might even necessarily be, just any arbitrary number, because a over b those are the two numbers that approximate e. If you think of them in terms of the reduced fraction, then a and b are unique, so it could be three, but you don't necessarily know that.

L: But he started with –

Mi: But in order to contradict –

L: I know what you're saying, but he's starting with a over b and then he's going to b factorial. See that's probably, the 2a, is probably the part that you're not getting, is that it's a times b minus 1 factorial is still an integer –

Mi: -- Yeah, but it's not proving for every integer, it's just proving for integers of that form.

L: No, no, no. B doesn't have to be of any particular form because you are starting with b and going to b factorial. You're not starting with b factorial and going to b.

S: So Ryan has this nice argument that if e equals a over b, so if e equals a over b then b factorial e is an integer. So the contrapositive of that is if b factorial e is not an integer then e does not equal a over b.

(Pause)

C: It may be helpful to think about this in the following sort of real world terms here. Suppose that it could be written in the form a over 3, then would it not be true that e could also be written in the form some integer over 3 factorial, by just multiplying the numerator by two?

M: Oh, I see (moves quickly towards the front of the room). Okay, so this is a way to think about it. Suppose that e could somehow be written as some number a over 5. Then it is also true that e is equal to a times 4 factorial divided by 5 factorial. (Mike moves back to the back of the room).

The dialogic discussions were characterized by a high level of student interaction. This translated into a large proportion of codes involving students making follow-up questions or comments. For example, on line 20 the student Mi follows up his original question responding to what the student L answered in line 17. Although he is also “responding to another student” this response is characterized by a need to resolve the prior question so it is considered a follow-up comment rather than just a response to another student. Compare this to the student L in line 17 responding to Mi’s question and whose statement has no other agenda than responding to the question of the student Mi. She does not attempt to raise any new issue or question that needs to be answered or dealt with. When Mi responds to her statement his only option is as a follow-up to his previous question. However, in line 23 the student L’s response of “No” is not considered a follow-up to her previous comment because it is a response to Mi’s follow-up rather than an obvious continuation of the issues raised in her previous comment.

We also see on line 38 the same student Mi following up his previous question with an additional comment. On line 41 we see the student L responding to Mi’s follow-up comment. This response was coded as both a student responding to student as well as a follow-up comment because of the content of what L says here and in her previous comment on line 34. On line 34 you see her state that he is starting with b and “then he’s going to b factorial”. She repeats this on lines 41 and 42 saying “because you are starting with b and going to b factorial". This connection between the content of her comments shows her following up her previous comment with this one rather than simply responding to Mi's comment.

The frequency of follow-up comments and questions found in this discussion is indicative of the *dialogic* discussions. This type of behavior in a discussion allows for the participants to resolve the issues that have been raised (generally by students) in the conversation because students are continuing to bring up these issues over and over until they are resolved. If issues were easily resolved or pushed aside then there would be very limited instances of follow-up comments because there would be no need for them. This shows that the *dialogic* discussions likely occur when there are difficult concepts in the content of the theorem or proof.

In addition to the high proportion of follow-up questions and comments made in *dialogic* discussions, there was also a high proportion of students responding to each other and asking questions of each other. There are several examples of this in the discussion from November. On lines 12, 17, 23, 30, 32, and 41 there are clear examples of students responding to the comments and questions of their fellow classmates. On line 12 the presenter R responds to Mi's question saying he doesn't understand what was said. On lines 17 and 23 the student L responds to Mi's question and follow-up comment. In line 32 Mi responds to L's comment with a new angle on his original issue rather than following-up his original question.

On the low end, there was a very small proportion of student behaviors that involved questioning the instructor, responding to the instructor, or making comments that were not related to the previous student comment. In the November discussion this did not occur at all. Overall in the *dialogic* discussions these behaviors accounted for less than 6% of all the student codes. In fact 94.1% of all the student codes during the *dialogic* discussions were related to a focus on students as active participants in the discussion, even if some had lower levels of student interaction associated with them.

In the November discussion there were very few instances of the instructor participating in the discussion. This is typical of *dialogic* discussions. Overall the instructor codes were only 11.1% of all codes. The most frequent of the instructor behaviors in the *dialogic* discussions were rephrasing student comments and questions, asking student questions, and making comments. If the instructor codes are grouped based on some commonalities we see that 23.3% of the time the instructor was participating he was asking students questions or generally asking for questions or comments. In the November discussion this did not occur. In addition, 21% of the time the instructor participated in the *dialogic* discussions he was asking a student to clarify or rephrase their or another student's comment or question. Once again this did not occur in the November discussion.

Although it appears as though the instructor behaviors in the November discussion do not reflect the more frequent behaviors in the *dialogic* discussions overall, we have to keep in mind that the fact that the instructor participated very little is consistent with the *dialogic* discussions. In terms of interrupting the discussion, this only occurred during 7% of all instructor codes. However, it did occur more in *dialogic* discussions than most of the other discussions. Therefore in line 56 of the November discussion the instructor, *M*, interrupting the discussion is reflective of a behavior that was more frequent in the *dialogic* discussions than the other discussion types.

Most *dialogic* discussions were initiated by students (67.7% of them). In the case of the November discussion we see in line 1 the instructor *M* calling on a student with a question. This discussion would be considered to be initiated by the instructor since he is the one who called on the student Mi to speak. In most of the *dialogic* discussions the student Mi would have begun the discussion by asking their question without prompting from the instructor. In addition, this discussion ended with a code that had a focus on the instructor rather than the students as active participants. We can see in line 45 the student S doing a nice job summarizing the presented proof and responding to the issue raised by Mi that is being discussed. However, rather than allow this student to continue to try and resolve the issue the TA, *C*, interjects with an example and then the instructor interrupts with a summary and example to resolve the issue. Once again this is not typical of the *dialogic* discussions. Only 32.3% of these discussions ended in a manner with a focus on the instructor. However, it is important to note in the November discussion that at least two of the students involved, L and S, recognized the issue that Mi had and offered good explanations that showed they were able to resolve this issue.

6.3.2 Student-Centered discussions

The *student-centered* discussions were the most frequent kind of student-to-student discussions that occurred in this class. These discussions occurred almost twice as often as any other kind of discussion in the continuum (see Figure 2). Some of the distinguishing characteristics of these discussions are the increased participation by the instructor, almost twice as much as in the *dialogic* discussions, and the more equal distribution between student comments and questions versus students responding to each other. In addition there are fewer instances of students making follow-up comments or questions.

The following example of a *student-centered* discussion is taken from late October. It is a discussion of a calculation using modular arithmetic.

(*M* is standing at the front of the room, having just made a statement regarding the use of notation in the problem)

A: I'm confused where when he used the second part, or the second time he used theorem 1.15 where he took away the squared from the 81 and the -1. Why wouldn't you do the same thing to the 2^{20}? Like raise it to the one half.

(*M* moves towards the back of the room)

M: R?

A: I don't understand how you could do that.

R: Wait, say that again.

A: Okay, on your third part right there, the second time you use theorem 1.15, how you have 81 squared and then -1 squared, then you have, right below that you have 81 and -1, without the square. Why wouldn't you do the same thing to 2^{20}?

R: Oh, yeah.

A: Because you show why 81 squared is congruent to -1 squared.

(Several students are talking over each other)

R: Well, I think I went about this a little backwards. Um, let's see. 81 squared is equal to -9 to the fourth, right, and so I was just trying to show where this would have come from. Um.

A: Oh.

M: So what would be the better order to put that?

R: Yeah, um. So show, well let's see. (Pause).

O: I also have a question. Like could you, like she said, could you put 2 raised to the 10 congruent modulo 81, mod 41, congruent modulo -1 mod 41. Could you put it all together since you know it's got a power of 2? You know 2 times 10 is 20. And since you have everything is squared.

A: I think I'm wanting to treat the congruence sign –

M: -- Ok, wait, wait, ok. So we have several questions here, let's pull them apart. So O has a question.

O: Okay.

M: O's question is, go ahead.

O: So you know how you have 2^{20} is congruent to 81 squared, since you can break up 2^{20} to 2^2 times 2^{10}, could you on the next line where you don't have the squares could you use 2^{10}.

R: Well, like this?

A: Right there.

O: Yeah.

R: Do we know that's true?

O: Yeah, that's what I'm saying. Does it follow? Cause that guy is saying no.

L: Yeah, it would but –

M: -- Ok, wait, wait, wait. Let's let R answer the question.

R: Uh. (Pause). Well, from up here if we just square both sides we'd get 2^5 squared equals -9 squared which would equal this.

O: So it would be?

(Pause)

O: Somebody else had a question?

M: Okay, so let me uh ask O's question in a more general way. (M moves to the front of the room) Can you formulate, in fact O can you formulate a general theorem that you are asking about?

The *student-centered* discussions were characterized by a lower level of student interaction than the *dialogic* discussions. While students are still responding to each other almost a third of the time they are participating, they are making general comments and questions almost as frequently. Most striking is that this is equivalent to the proportion of the time that the students ask questions of each other plus the proportion of time they made follow-up comments or questions.

We can see in line 1 that A's question doesn't appear to be directed at anyone in particular. He starts out saying he is confused "when *he* used the second part". Notice he doesn't say "*you*". This implies he was not directing his question to the presenter. It is not until the presenter asks him to say his question again that A directs his question to the presenter: "Why wouldn't *you* do the same thing to 2^{20}?" (line 16).

There were fewer follow-up comments in this discussion than the *dialogic* discussions. In line 10 A rephrases his question after the instructor redirects this question to the presenter, but the next follow-up comment or question doesn't occur until line 61 when the student O follows up on his earlier question from line 49.

There is a higher proportion of time spent questioning and responding to the instructor as compared to the *dialogic* discussions. This still represents less than 7% of all the student codes; however we do see a student respond to the instructor in the

October discussion. On line 33, the student R responds to the instructor's question about the order his proof should be in. There were no examples in this discussion of students questioning instructors, however this did account for 1.3% of all student codes in these discussions.

One of the most interesting student codes in the *student-centered* discussions is the SN code which represents a student making a comment or asking a question that is not related to the previous student comment. The proportion of time this occurred in the *student-centered* is the second highest among all seven discussion types discussed in the previous chapter. We see this in line 35 where the student O asks a question which at first seems unrelated to the previous student comments and questions. It appears that he is asking if he can put a string of congruencies together because they are all squared. It appears as though the original student A's question has been left unresolved and a new direction has been taken in the discussion. It isn't until line 49 when the instructor has O repeat and clarify his question that we see that their questions are related. So although there are higher numbers of the SN code in these discussions often these come from students unable to clearly link their concerns with the issue that has been raised by a classmate.

Overall, we see that the student behaviors are different in the *student-centered* discussions than they were in the *dialogic* discussions. While there was still a high percentage of time being spent on behaviors that are indicative of students being active participants in the discussion, this was lower. Only 87.7% of the time students participated did they participate in this way. In addition students are asking as many new questions to each other as they are making follow-up comments and questions. The biggest change is in the SN codes, or how often students make comments or ask questions that are not related to the previous student comment. Although there is a high level of interaction and student participation in these discussions, this participation is less focused on responding to each other and continuing the discussion in the same course until your issue is resolved.

We can see in the October discussion that the instructor participated more often than he did in the November discussion. Although we didn't see it in the October discussion, in the *student-centered* discussions the instructor asks for comments and questions more frequently than in any other type of discussion. In addition the proportion of time spent asking students to clarify or rephrase themselves is about half what it was during the *dialogic* discussions. We do see this occur once in the October discussion. On line 47 the instructor asks the student O to repeat his question.

However, what is most interesting is the combination of the PRP code, the PQ code, the PI code, and the PF code in this discussion. All three seem to indicate the instructor's role in these discussions as the moderator making sure that the students are answering each other's questions, keeping the focus on questions he believes need to be asked, and deciding who will respond to some questions. This behavior is fairly limited in this discussion (and all the *student-centered* discussions) but is more apparent than it is in the *dialogic* discussions. On line 8 the instructor directs a question to the presenter and does this again on line 66 when he interrupts the student L to ask R, the presenter, to respond to O's question.

More than fifty percent of the *student-centered* discussions were initiated by students. This is a bit less than the *dialogic* discussions; however it is still a majority. In this particular discussion the student A initiated the discussion by asking a question about the presented proof. All but one of the *student-centered* discussions initiated by students began with either a student comment or a student question. Those that were initiated by a student question were equally split between questions directed to other students and general questions. This is higher than in the *dialogic* discussions where almost all discussions initiated by students with a question were questions clearly directed to another student.

The ends of the discussions were split almost equally between a focus on the instructor and a focus on the students as active participants. In the *dialogic* discussions the student-focused ends were approximately two-thirds of all discussion endings. We can see this distinction in the October discussion since the discussion ends with the instructor directing the next course the discussion will take. It is clear that the instructor sees a general theorem that relates to what is being discussed by the class; however he points the class in this direction, rather than allowing a student to do so. We can also see that in this discussion it is not clear that the issue that was raised by the student A in the beginning of the discussion has been resolved. At one point it appears as though the student L (line 64) believes that it is okay to take the square root of both sides of a congruence. This is more indicative of the manner in which the *student-centered* discussions ended; in fact about one-third of the discussions ended with the students having resolved the issue being discussed.

Generally, the *student-centered* discussions contain student interaction. However, the interaction appears disjointed and sometimes completely unconnected as seen by the high proportion of SN codes compared to the other discussions. The students respond to each other as often as they make general comments and questions and the instructor plays a bigger role in directing the course of the discussion.

6.3.3 Socratic discussions

The *Socratic* discussions are characterized by a larger proportion of instructor participation than in any of the other four main discussion types. In addition, the instructor played the role of the authority more often in these discussions than the other four main discussion types. The students were only participating with high levels of interaction one fourth of the time that they were participating. In the following transcribed discussion from late September notice the number of utterances that are made by the instructor compared to the students. Also notice how often the students respond to each other versus responding to the instructor.

(*M* is standing in the front of the room)

M: So let's discuss why we would want to define prime in that way. What's an alternative definition that you might consider?

O: Only divisible by one and itself.

M: Yeah, only divisible by one and itself. That's a natural thing that you might say.

L: But then one is kind of prime.

St: Cause one is only divisible by one and itself.

O: One is what?

T: Then you just define 1 not to be prime.

M: One would be prime. So the question really comes down to should one be a prime.

O: So what is one? I know it's not prime, it's not composite. Does it have a, does it have a name?

M: So the question is, the question is should we define one to be a prime or not. This is really the same kind of discussion we were just having. Is it useful to have one as a prime or to not have one as a prime and what's the reason for choosing it one way or the other if there is one, or is it just arbitrary. Yeah, D?

D: When we get to the unique factorization theorem later, having one be a prime will really screw that up.

M: Okay, why?

D: Because then you can just express anything as whatever you would express it times one to whatever power you want.

(**L** is talking over **D** during this)

M: So in other words, unique factorization would not be a theorem any more.

D: Right.

M: It wouldn't be if Da factored the number six he might get one thing and you know Ma factors six and gets a different answer. Because Da could get 2 times 3, Ma could get 2 times 3 times 1 times 1. So you wouldn't have unique factorization into primes if you allowed one to be prime. So that's a good reason for having the definition of prime not include 1 as a prime.

The *Socratic* discussions have several distinguishing characteristics in terms of student participation and the role they play. The students are still participating more often than the instructor is, however these two are much closer in proportion than they are in any other of the four main discussion types. In addition, when the students are participating they are only participating in ways that involve high levels of interaction between students less than one-fourth of the time.

The most frequent behavior the students engage in during the *Socratic* discussions is responding to the instructor. This fits with the description of these discussions as having limited interactions between students with most interactions being between the instructor and students. In the late September discussion we see how often the students respond to the instructor. In this discussion there were four student utterances that were coded SRI. These occurred on lines 4, 29, 34, and 42. The three on lines 29, 34, and 42 are all the same student, D, and really demonstrate the nature of a *Socratic* discussion where there are back and forth interactions between the instructor and the students.

Another common behavior of students during the *Socratic* discussions is the high number of comments and questions that are not directed to anyone in particular. We see in lines 13 and 20 the student O doing this. In addition on lines 9 – 15 we see students making comments. These comments do appear to be extensions of the prior students comment (so they are related) however, they are not really responding to each other because they are not acknowledging the earlier comment they are just making statements that while related can be taken on their own.

The instructor's role in these discussions is characterized by a high level of questioning as well as commenting and acting as an authority. The instructor's utterances also constitute 41.5% of all the utterances. This is the highest of all the four main discussion types (even though *univocal* discussions are just one percent less). In terms of instructor behaviors that focus on students as active participants, there are two behaviors that occur twice as often as any others. These are the PFQ and PQ codes, the instructor asks a follow-up question based on a student response and the instructor asks a student a question. We can see that in 4 of the 7 utterances the instructor makes in the late September discussion he is asking a question of some kind. He initiates the discussion in line 2 by asking the class a question. In line 17 he asks the newly re-focused question to the whole class and in line 23 reiterates this question rather than responding to the student O's question or allowing a student to respond to his question. Then in line 32 he asks the student D a follow-up question based on his response to the instructor's earlier question.

The only other instructor behavior that focuses on students as active participants which occurred with some frequency is the PR code; the instructor rephrases a student's comment or question. We can see on line 39 that the instructor has rephrased the responses of the student D from lines 29 and 34. This did not occur as frequently as the questioning the instructor did, which made up over 60% of all the behaviors focused on students, however this was the next most frequent behavior.

The two instructor behaviors that focused on the instructor that occurred most often in *Socratic* discussions were the PA and PCM codes. These made up almost 75% of all the codes that reflected a focus on the instructor and nearly 40% of all the instructor codes. We can see an instance of the PCM code (instructor makes a comment) in line 17 when the instructor states that "one would be a prime". In addition, this can be seen almost as the instructor acting as an authority by making such a definitive statement compared to the student L's statement in line 9 "but then one is kind of prime". We see another example of the instructor commenting as an authority during the final comment of the discussion. In line 46 he states "so you wouldn't have unique factorization into primes if you allowed one to be prime". This is somewhat of a rephrasing of D's earlier comments, however he then goes on to say "so that's a good reason for having the definition of prime not include 1 as a prime" rather than allowing the students to judge this explanation that D gave.

The remaining characteristics of the *Socratic* discussions come from the initiating and ending of the discussions. In terms of initiating discussions, the instructor initiated nearly 65% of the time. This is almost twice as often as in the *dialogic* discussions. We can see that in the late September discussion the instructor was the one who initiated it making this particular discussion reflective of most *Socratic* discussions. Out of the 33 discussions, only 13 ended with the students having resolved the issue. In this particular discussion D resolved the issue with his comments in lines 29 and 34. However, we also see that the instructor chose to acknowledge this by rephrasing this explanation and offering an example. In addition he served as more of an authority by offering a judgment on the reasoning that D offered rather than simply acknowledging D's explanation. Overall, the *Socratic* discussions ended with a focus on the instructor more than half of the time. The focus was on the students at the end of only 42.4% of the discussions. As we saw in the late September discussion, even when the students (or a student) had resolved the issue, the instructor still displayed some behaviors that were reflective of a focus on the instructor.

6.3.4 Univocal discussions

The *univocal* discussions were described to have no give and take and that students did not respond to each other. In discussions that involve at least two students it is difficult to find discussions that involve absolutely no student-to-student interaction. However, this probably explains why there were only 12 examples of these discussions. There were two of these discussions that were the instructor asking a question, several students in the class responding, and the instructor making some statement to end the discussion. The other ten discussions contained very limited, if any, moments when students responded to each other.

In the following discussion from mid September we can see the limited nature of the student interaction. There is some responding to each other; however it is generally to clarify what the other person said rather than actually responding to what the person is saying. Notice how much the instructor participates in the discussion and that most of the statements made or questions asked by the students are quite brief.

T: The subscripts on r, 1 and (m+1), does that also cover m? Like, does it start from i, and then i becomes your m, and then m+1 is equal to the m that was referred to in the assume for all 1 is less than or equal to i is less than m?

(Pause)

T: So you're showing the plus one case.

R: Right.

T: So does m equal to i in that?

Mi: That varies from 1 to m.

T: No.

M: So let me rephrase his question, T's question. Or let me ask a question that I think will answer T's question. Namely, when you say assume for 1 less than or equal to i less than or equal to m, $r_i = x_i a + y_i b$.

R: Right.

M: Suppose instead of writing it in that compact form could you write down a series of equalities that you assume that you have found. What would they be?

(Pause)

T: Well in this proof everything is equal to 1 through m minus 1. I'm just asking if it goes up to m minus 1, and you wrote down m plus 1, does it also cover m?

M: So you're asking whether that is a less than or a less than or equal sign.

T: Yeah.

M: Is that a less than or a less than or equal sign?

R: Well I have it as less than or equal.

M: Yeah, that's correct, so put that down, less than or equal. Make it so it's more clear on the screen.

R: It includes m.

M: It includes m, so you're assuming it's true for m. You've already shown that rm is equal to a linear combination of a and b.

The high proportion of time that students responded to the instructor during the *univocal* discussions can be seen on lines 22, 30, 37, and 40. On all four of these lines the students respond to a question from the instructor. In the case of the student T on line 37, the response is an attempt to repeat his question – no one appears to have an answer to the instructor's question on line 31. The other three times all consisted of a brief response to the instructor's question. After the instructor entered the discussion the students did not have any other substantial contributions to the discussion beyond responding to the instructor.

The instructor codes that focused on the students and occurred more often, in comparison with the other discussions, included the PR, PFQ, PQ, and PQC codes. In the mid September discussion we see examples of three of the four, further supporting this observation. On line 17 when the instructor enters the discussion we can see that he is rephrasing T's question. Notice that the instructor chooses to rephrase the question rather than have the student try to clarify. The second time the instructor speaks, line 23, he asks a follow-up question based on his statement in line 17. He asks another follow-up question on line 33 after the student T repeats his question. The fourth time he speaks in line 37 he asks a question of the presenter. The only other time the instructor speaks (line 41) before the end of the discussion was coded as PA (the instructor answers a student's question or addresses a student's comment directly – acts as an authority) which is a code that focuses on the instructor and occurred one-fourth of the time that the instructor's behavior had a focus on the instructor rather than the students.

In terms of initiating the *univocal* discussions, the September discussion is one of only two that were initiated by students and therefore is not indicative of most *univocal* discussions. However, both of these discussions were initiated by the students asking a general question which was not directed at anyone in particular which is in contrast to the other three main discussion types.

Only two of the *univocal* discussions ended after the students had resolved the issue that had been raised. Generally the instructor took control and re-directed the discussion, began a discussion with a single student, or acted as an authority and made a comment or mini lecture. In this case the discussion ended with the instructor telling the student how to correct his proof and saying why this was necessary.

7. Conclusions

The discussions that involved two or more students in this particular inquiry-oriented transition to proof course fell along a continuum based on the level of student interaction – from *univocal* (i.e. monologic) to *dialogic*. This continuum was loosely designed based on the work of Knuth and Perressini (2001) and is supported by sociocultural learning theory, in particular the work of Vygotsky and Yakubinsky (Wertsch, 1985). The original design did not offer very detailed descriptions of these discussions, just enough to be able to place all the discussions at a point along the continuum. I found that there were a fair number of discussions in my data that did not fit nicely into one of the four main discussion types that I defined during the initial analysis. In the previous chapter I chose to focus only on the data from the four main discussion types in order to provide more detailed descriptions of these in the revised continuum (see Figure 5). A possibility for future research is to develop deep descriptions of these in-between discussion types in the hope of further revising the continuum of these discussions in inquiry-oriented classrooms.

This continuum is meant to answer my research question: What are the roles of the instructor and students throughout a semester in the discussions that occur in an inquiry-oriented course where the roles of proof to communicate and explain mathematics are valued as integral parts of the proof construction and validation processes? The continuum shows these different roles of the students and instructor and how these varied among the four main types of student-to-student discussions that occurred throughout the semester.

There are several key conclusions that can be drawn from my analysis. The first of these involves the analysis of the discussions based on the difficulty of the content being discussed. The data clearly show that topics that are considered to be the more difficult content in the course are more frequently the content of *dialogic* discussions. This is in opposition to a widely held belief that difficult mathematics content needs to be "delivered" to the students by the knowledgeable expert, the teacher (Stroup et al., 2007) Although this research focused on discussions between students and does not include instances of *univocal* discussions that only contain the instructor, it does show that students are capable of talking about difficult content in a highly interactive way and based on the research of Smith (2006) we can see that these students tend to have a more well-developed sense of mathematical proof, which is a goal of undergraduate mathematics courses.

One of the characteristics that I found in terms of the roles of the students as well as the instructor in each of the four main discussion types is consistent with sociocultural learning theory and the distinctions made between dialogic and monologic speech. Yakubinsky argued that monologue speech is linked to the idea of an authority (Yakubinsky & Eskin, 1997). This is consistent with the *univocal* discussions I found in this particular classroom. Generally, during these discussions either the instructor or a student served as an authority and those responding made brief conciliatory responses. However, I also found an interesting characteristic in these discussions that doesn't necessarily fit with the sociocultural learning theory I presented. Both Yakubinsky (1997) and Stroup et al. (2007) make the argument that with dialogic speech fewer words are needed because of the shared apperceptual mass. In my analysis, I found that in *dialogic* discussions there were more utterances per participant than any of the other discussions. Although this is in contradiction to the theory, it may be that these utterances were shorter; however I have not conducted an analysis on the lengths of these utterances.

Another important conclusion stems from the types of discussions and how they were distributed throughout the semester. One might expect that the participation of the students, and in particular the level of interaction between them, would increase as the semester progressed and the participation of the instructor would decrease. Wood (1991) found this with her research. This would imply fewer *univocal* discussions at the end of the semester. Stroup et al. (2007) argued that with novel mathematics ideas the students tended to participate in less dialogic discussion until the growth of the apperceptual mass occurred. This would imply more *dialogic* discussions at the end of the semester than at the beginning.

However, neither of these occurred. In fact all four types of discussions were present throughout the semester, and *univocal* discussions were at their lowest in the middle of the semester, not at the end. In addition *dialogic* discussions were at their highest in the middle of the semester, not at the end.

The most important conclusion that can be drawn from this research is the implications for teacher preparation. The notion of inquiry-oriented teaching as being hands-off and lacking in terms of teacher talk have been contradicted by this research. In this classroom, as shown by the variety of discussions occurring throughout the semester, it is clear that the instructor played an active role in discussions throughout the entire semester. However, what is most important is to understand the different roles the instructor played in response to the roles the students took on in the different types of discussions. It is also important for future inquiry teachers to understand that there are many types of discussions that occur in a mathematics classroom where student artifacts are the focus of the class time. Although this research was conducted at the undergraduate level and proof is the focus of the discussions, I believe the conclusions found here can be translated into other mathematical inquiry-oriented classrooms where forms of mathematical proof generated by students are discussed.

The literature on proof supports the social manner in which proof was learned in this particular class (Hersh, 1993; Hanna & Jahnke, 1996). In addition one of the roles of proof is to explain and bring about understanding. This understanding ties into the need to study the discourse in this type of class where proof is being learned. Much of the research on mathematical discourse argues that there is a direct link to participation in mathematical discourse and understanding (Sfard & Kieran, 2001; Wood et al., 1993; Yackel & Cobb, 1996). However, much of the prior research did not offer a complete picture of the types of discussions that occur and what role the students and instructor play in these discussions.

My dissertation research offers insight into the types of discussions that occur in an inquiry-oriented course that emphasizes students' mathematical artifacts – proof at the undergraduate level – and the roles of the students and instructor in these discussions. While I have not addressed what this means in terms of student understanding, the prior research on discourse has argued that these discussions with more than two students involved, and especially those with higher levels of student interaction, likely increase student understanding.

7.1 Limitations

Although I believe this research is useful it also has limitations. The most significant of these is that this research only studied one class for one semester. Thus, generalizing the results of this research and transferring these to other teachers and other classes may be premature. Since many of the behaviors of both the instructor and students were based on actions of the other students and instructor and this exact class can never be duplicated, the results of this research are unique. However, the types of discussions that occurred in this particular class and the roles of the students and instructor in them can be used to study other classes with the understanding that each classroom situation is different.

A second limitation involves the data collection. The class was not videotaped every day. In addition, the video focuses on the instructor, which limits the data in terms of how well it captures the atmosphere of the class and student behavior. The original research question was focused on the role of the instructor and only widened to include the role of the students during particular discussions after analysis of the video began. Although the video focuses on the instructor it was able to capture what was said by the students.

A final limitation results from the data analysis. The manner in which I chose to summarize my video data and the actual summaries I wrote could have been limiting.

Transcribing all the video data would have been overwhelming, so summaries were used, which cannot include every detail of what occurred in each class. Also, when I generated codes, these were based on my observations of the video and may be limited.

However, I recognized these limitations from the outset of my research and I attempted to balance these biases and to be as thorough with my data collection and analysis as possible. These limitations do not affect the relevance of this research, but may limit how broadly its conclusions can be applied.

7.2 Future Research

Further questions that may arise from this research include: What roles do the instructor and students play in the in-between discussions? What do the discussions that involve two or more students look like in other inquiry-oriented proof courses with other professors? Do these discussions in courses taught by the same instructor vary from semester to semester? In addition, it would be of interest to take the continuum and use it to analyze these discussions in other classes in order to revise it so that it may be applied more broadly to inquiry-oriented proof courses.

Appendix I – Coding Scheme

	Initiated by?	Role of Students	Role of Professor	End of Conversation	Role of TA
FOCUS ON STUDENTS (PLURAL) BEING ACTIVE PARTICIPANTS IN THE DISCUSSION	**IP**- initiated by presenter **IS** – initiated by student (not presenter)	**SC** – student makes comment **SQ – s**tudent asks question – unknown to whom **SQP** – ask question of presenter **SQS** – ask question of another student **SRP**- respond to question/comment of presenter **SRS** – respond to question/comment of another student **SF** – make follow-up question/comment after previous question/comment **SPF** – presenter facilitates discussion by calling on students	**PC** - asks student to clarify or repeat their comment/question **PR** – rephrases a students comment or question **PFQ** – asks a follow-up question based on a student response **PQ** – asks student or presenter a question **PG** – has students work in groups **PRP** – redirects a student question/comment to another student or presenter **PP** – acts as a peer, contributing equally in the discussion **PQPI** –asks presenter question about proof (incorrect) **PQC** – asks class for questions or comments **PN –** asks students to give their names	**CS** – issue is resolved by the students and Instructor /TA responds to this **CML** – Instructor interrupts a discussion that contains limited student participation to rephrase the issue/comme nts being made **CMRd** – Instructor interrupts the discussion to redirect the class **CST** – issue is resolved by students	**TCl** - asks student to clarify or repeat their comment/ques tion **TR** – rephrases a students comment or question **TF** – asks a follow-up question based on a student response **TQ** – asks student or presenter a question **TP** – acts as a peer, contributing equally in the discussion **TQPI** –asks presenter question about proof (incorrect)

FOCUS ON INSTRUCTORS	**IM** – initiated by Instructor **IT** – initiated by TA	**SQI** – ask question of Instructor **SRI**- respond to question/comment of Instructor **SN** – question/comment is not related to previous STUDENT comment	**PS** – summarizes or rephrases the presented proof **PSc** – scaffolds a student through a presentation by repeated questioning **PF** – facilitates the discussion by calling on particular students **PA –** answers a student's question or addresses a students' comment directly – acts as an authority **PI** – interrupts discussion or proof presentation **PCM** – makes comment	**CMR** – Instructor interrupts a discussion to rephrase the arguments given **CMSS** – Instructor interrupts the discussion and begins a discussion with a single student **CMS** – gives summary of proof – not an interruption **CT** – time runs out **CL –** lecture or make statement	**TSc** – scaffolds a student through a presentation by repeated questioning **TA –** answers a student's question or addresses a students' comment directly – acts as an authority **TI** – interrupts discussion or proof presentation **TC** – makes comment

Appendix II – Initial List of Student and Instructor Behaviors

MARK

1. ASKS QUESTION

asks for questions after a presented proof
gives option for comments in addition to questions after a presented proof
asks for questions/comments after a discussion of a presented proof
asks for further comments
questions class
asks class for improvements to presented proof
questions class on part of presented proof
questions presenter
asks student for clarification
asks presenter for clarification
asks students to evaluate a presented proof
asks class for clarification
asks class if they understand presented proof
asks student to make connection with previous proof

2. DIRECTS DISCUSSION

initiates discussion (non-verbal)?
directs students to ask their questions directly to the presenter
calls on a student with a question
calls on a student with a question (several have hands raised)
calls on student(s) with comments
serves as mediator between student and presenter
directs attention back to proof (wants to understand what proof says)

3. OTHER

asks presenter to decide to reject or accept changes
confirms/agrees with student response
asks students if he has summarized correctly
asks if class agrees with his evaluation
has students talk to each other about proof

4. CLARIFES/EXTENDS/REPHRASES

extends student comment
interjects with clarification
rephrases/summarizes student question
rephrases/summarizes student comments
summarizes/rephrases presented proof

STUDENT/CLASS

Student(s) to presenter discussion

Student question to Mark regarding assumptions
Student question to Mark regarding administrative issue
Student question to class
Class responds to student question
Class responds to Mark question
Student comment on presented proof
Student(s) to student(s) discussion
Student to Mike discussion

PRESENTER
Presenter calls on student with a question
Presenter asks for questions after a presented proof
Mike – Presenter discussion (question and answer)

Appendix III – Descriptions of Six Students

LISA[14]

Lisa has an amazing ability to detect problems in the details of a proof, a skill that probably has a lot to do with her background as a CS major. She goes out of her way to address these potential flaws, rather than trying to wave them off. Her homework was generally quite neat, and written in meticulous detail, but the level of detail generally didn't obscure the main ideas behind what she was writing down. Her in-class exams were a little more problematic; unlike a lot of her classmates, she usually seemed to know better than to write complete nonsense, but she did forget some things when under time pressure. In class, Lisa did an amazing job of setting a good tone for the class - she was the most vocal person in the class on the first day, and she never really let up. She was very up-front about things and didn't mind telling you if you were wrong about something. I think some of her classmates interpreted this as one-upmanship, and more than a couple of people told me that they found her a bit intimidating (although they didn't use that word). I don't think Lisa was really aware of this, although I could be wrong. Based on what she brought to the class, Lisa is the one person I couldn't imagine running this class without.

SUSAN

Susan is nothing if not hard-working. I deeply appreciated the effort she put into this class even though it was outside the main focus of her degree program, which is in actuarial math. On Theorem 3.15, one of the most demanding theorems we ask the students to prove, Susan turned in several proofs, in succession, that were incorrect (for various, subtle reasons). Rather than quitting (as most students did on this problem), Susan continued turning in proofs until she produced one that was perfect. I think that says a lot about her character, since she probably would have gotten a greater marginal benefit (in terms of her grade) by working on other theorems. Susan struggled a lot on the in-class tests; she didn't seem to have enough of a command of the material to be able to recall important ideas quickly. However, she did a tremendous job on the take-home exam, much better than I could have anticipated. Susan's written work was generally quite neat and easy to read; I don't remember her hand waving a lot, or trying to get away with nonsense. I believe she ended up with a C in the class, but based on her effort, one could have made a strong case that she deserved a B.

ALLISON

Allison made a C in this course, but I don't feel that she deserved more than a D. Her attendance in class was sporadic, and she didn't make many presentations towards the end of the semester. Her homework was generally very neatly written, but unfortunately a lot of her proofs failed to really prove anything, and I got the impression at times that she wasn't able to tell whether an argument that she wrote was correct or not. Allison usually wasn't able to do very much on the exams, especially the ones that were given in class,

14 The names of the students have been changed throughout this appendix.

where she didn't have the ability to look back at her notes. In a conversation we had around the middle of the semester, Allison told me that she desperately needed to pass this course, since she was bound by contract to graduate this year so that she could enter the Air Force. At that time, I tried to give Allison a realistic idea of what she needed to do in order to make it through, and that included (among other things) much steadier class attendance. I never got the impression that she made a particular effort to hold up her end of the bargain.

DAVID

Some other students have heard me say that talking about mathematics with David is sort of like popping several caffeine pills, and then trying to read a math book. It's a bizarre experience, it's hard to focus, and there's no guarantee that you'll be any wiser for the experience, but hell if it isn't fun. David earned my respect, as well as Dr. Starbird's, by discovering a proof of the divergence of the series 1/p_n that neither Dr. Starbird nor I had thought of. This proof was based on an idea that is so elegant that it can be explained to even a beginning student, and the fact that David had the vision to see this approach says a lot about his potential as a mathematician. David tried to present his proof on the last day of class, but under the time constraints he was working under, he was unable to gather his thoughts and make a good presentation, which I thought was a tragedy considering how pristine his proof was. David's proofs often have a very Spartan quality to them - he doesn't waste a lot of words, he gets right to the point, and sometimes he makes a leap of logic that you have to think for a minute to process. By talking to him, one finds that this isn't something he does for the sake of appearances - David's mind really does work that quickly, and if you aren't pretty quick yourself, you'll have a hard time keeping up. David didn't make a whole lot of comments during class, but I got the impression that he was holding back a little, waiting until he had something to say that he knew that other people weren't already going to say.

MARK

Mark did some things during the semester that impressed me, and one thing that really tarnished everything else for me. There was one theorem, in Chapter 2, that he didn't know how to prove directly, so he went through a couple of lemmas en route to discovering a proof of the theorem. I was impressed with his persistence in doing this, and it really was one of the most mathematician-like things I saw anyone do all semester. Mark's written work was always very nice, neatly written, and easy to read. As the semester went on, he did get a little behind on his work (I got the impression he was taking a tough course load), and his attendance in class was inconsistent, so his participation wasn't quite what I hoped it would be. But from what I could tell, he didn't seem to have any trouble keeping up with the ideas presented in class. The thing that Mark did that made a terrible impression on me was this: on the take-home exam, there was a question about the Chinese Remainder Theorem. Mark turned in an argument that followed a proof that I was familiar with from a different number theory course I had taken - it was quite different from the one we used in class. And it wasn't the sort of argument you expect a student to come up with on his/her first try, because it requires a lot of insight about the multiplicative structure of the integers mod n, which we hadn't really set up yet. When pressed to explain how he came up with the idea, Mark admitted that he had found it in a book. He looked up the theorem because he had missed the class sessions when we talked about it, and he was under pressure to make a good grade in the class. For his transgression, Mark was given a zero on the take-home exam, and a C in the course. If his other work had not been of the highest quality, he probably would have failed.

KEITH

Keith was one of my favorite students because of the progress he made from the beginning of the semester to the end. At the beginning of the semester, his work was good enough, but not great, and he didn't seem particularly adept at solving difficult problems. By the end of the semester, he was doing very well on homework and exams. You could always tell when Keith was satisfied, proud of something he did, or frustrated, just from looking at his written work - he tended to annotate his proofs with statements like "I don't understand this" or "I don't know how to go on from here" or the like. He didn't try to cover up gaps in his understanding by writing nonsense, and he seemed to have a very good general feel for whether what he was writing made sense. And besides all of this, he was an unfailingly pleasant person, always polite in class, and seemed to value the comments of the other students when he was presenting

Appendix IV – Instructor Interview Protocols

Interview I Protocol

1. How would you describe your teaching philosophy?
   (jot down major points and make sure these are covered in 2)

2. How does the way you teach the Number Theory course exemplify your teaching philosophy?

3. This is a course which is intended for students to learn to prove. Why do you think this method of teaching serves this purpose?

4. How would you describe an ideal Modified Moore Method Number Theory course?

5. What would the role of the instructor be in this ideal course?

6. To what extent do you feel that the courses you have taught using MMM have fit this ideal?
   (in cases where he has gotten there, or really close, what does he think is the cause?)

7. What role do you feel you play in the actual MMM courses you have taught, and are teaching?
   (what is your "purpose" in the class…?)

8. How do you feel the semester has gone so far?
   (How would you describe your current MMM Number Theory Course?)

9. How would you describe your role in this course so far?

10. To what extent does your role change as the semester progresses in this kind of course?
    (if no, why do you think your role doesn't change?, if yes, 11)

11. What do you think affects how your role changes?

12. What do you expect for this semester?

13. Do you have any questions for me or any other thoughts about this?

Appendix V – Student Interview Protocols

Student Interview I Protocol

1. Describe your Number Theory, M328K, class. (*If student does not describe specifically how they view the role of instructor and themselves, ask)*

2. Overall has the role the instructor has played in this class remained consistent, both within a single class and over the semester, or do you feel it has changed over time? Explain.

3. How would you compare the role the instructor plays in this class to that of the average instructor in more traditional lecture-style courses you have taken?

4. How, and by whom, is it determined in class whether or not a presented proof is correct?

5. If you could name three things that are important descriptors of a good proof what would they be?

Final Student Interview

1. Please describe a typical class at this point (the end) of the semester. (Specifically what the instructor does)

2. During a class discussion what type of role does the instructor usually play? If this varies please describe the different situations and what role he plays in each, i.e. did he play a more prominent role at any point?

3. What does it mean to you to understand proof and/or Number Theory? What do you think it means to Dr. Starbird?

4. Earlier in the semester you described how a proof is accepted in class. How is a proof accepted in class as valid now? What part does Dr. Starbird play during this process?

5. In what ways do students work together in this class? Has Dr. Starbird been explicit as to when and how students should work together?

6. Overall, how would you describe the role Dr. Starbird has played in this class over the whole semester?

7. Would you take a course taught using the Modified Moore Method in the future? Would you take a class taught by Dr. Starbird in the future?

8. Do you have any questions for me?

Appendix VI – IRB Consent Forms

IRB# 2003-04-0084

Informed Consent to Participate in Research

The University of Texas at Austin

You are being asked to participate in a research study. This form provides you with information about the study. The Principal Investigator (the person in charge of this research) or his/her representative will also describe this study to you and answer all of your questions. Please read the information below and ask questions about anything you don't understand before deciding whether or not to take part. Your participation is entirely voluntary and you can refuse to participate without penalty or loss of benefits to which you are otherwise entitled. You will be given a copy of this form for your records.

Title of Research Study:

The Instructor's Role in the Development of a Classroom Community of Inquiry (contained within IRB protocol of 2003-04-0084)

Principal Investigator, UT affiliation, and Telephone Number:

Stephanie Ryan Nichols, M.S., Doctoral Candidate, Department of Curriculum & Instruction, (512) 232-3957

(Dissertation Supervisor: Jennifer Christian Smith, Ph.D., Assistant Professor, Department of Curriculum & Instruction, (512) 232-9682)

Funding source:

None

What is the purpose of this study?

The purpose of this study is to investigate the teaching of an expert mathematics instructor, more specifically the instructor's role in this development of a classroom community of inquiry.

What will be done if you take part in this research study?

You are being invited to participate in this study because you are the instructor for a section of M 328K in the Fall 2005 semester. Should you choose to participate, five students enrolled in your section will be invited to participate in the interview portion of the study, and a member of the research team will videotape the class two to three times a week. You will be interviewed approximately every other week during the semester to discuss your role in the development of the class, and your thoughts about the class as a whole. Each interview will take place on the UT campus at a time and a location to be agreed upon by you and the investigator. Each interview will last approximately half an hour. These interviews will be audio-recorded and transcribed.

What are the possible discomforts and risks?

Some people feel a certain degree of anxiety when being interviewed, recorded, and videotaped. We will make every effort to make you feel comfortable during the interviews. You are not required to answer every question asked of you.

If you wish to discuss the information above or any other risks you may experience, you may ask questions now or call the Principal Investigator listed on the front page of this form.

What are the possible benefits to you or to others?

There are no direct benefits to you for your participation. Future mathematics students and pre-service teachers may benefit from improved instructional techniques that may be developed from the results of this study.

If you choose to take part in this study, will it cost you anything?

No.

Will you receive compensation for your participation in this study?

No.

What if you are injured because of the study?

There are no foreseeable physical risks associated with this study.

If you do not want to take part in this study, what other options are available to you?

Participation in this study is entirely voluntary. You are free to refuse to be in the study, and your refusal will not influence current or future relationships with The University of Texas at Austin.

How can you withdraw from this research study and who should I call if I have questions?

If you wish to stop your participation in this research study for any reason, you should contact: Stephanie Nichols at (512) 232-3957 or (512) 773-7884 (cell). You are free to withdraw your consent and stop participation in this research study at any time without penalty or loss of benefits for which you may be entitled. Throughout the study, the researchers will notify you of new information that may become available and that might affect your decision to remain in the study.

In addition, if you have questions about your rights as a research participant, please contact Clarke A. Burnham, Ph.D., Chair, The University of Texas at Austin Institutional Review Board for the Protection of Human Subjects, 512/232-4383.

How will your privacy and the confidentiality of your research records be protected?

Authorized persons from The University of Texas at Austin and the Institutional Review Board have the legal right to review your research records and will protect the confidentiality of those records to the extent permitted by law. If the research project is sponsored then the sponsor also has the legal right to review your research records. Otherwise, your research records will not be released without your consent unless required by law or a court order.

If the results of this research are published or presented at scientific meetings, your identity will not be disclosed.

The interviews will be audio-taped and the cassettes will be coded so that no personally identifying information will be visible on them. You will be assigned a pseudonym, and you will be referred to using this pseudonym by the researchers when discussing the results of the interviews. The interviews will not be discussed with the student participants or with the other instructors. Cassettes of the interviews will be kept in a secure location and will only be heard by members of the research team. The cassettes will be transcribed and will be kept indefinitely for future reference and analysis.

Will the researchers benefit from your participation in this study?

No.

Signatures:

As a representative of this study, I have explained the purpose, the procedures, the benefits, and the risks that are involved in this research study:

Signature and printed name of person obtaining consent **Date**

You have been informed about this study's purpose, procedures, possible benefits and risks, and you have received a copy of this Form. You have been given the opportunity to ask questions before you sign, and you have been told that you can ask other questions at any time. You voluntarily agree to participate in this study. By signing this form, you are not waiving any of your legal rights.

Printed Name of Subject **Date**

Signature of Subject **Date**

Signature of Principal Investigator **Date**

Signature of Dissertation Supervisor **Date**

We may wish to present some of the tapes from this study at scientific conventions or as demonstrations in classrooms. Please sign below if you are willing to allow us to do so with the tape of your performance.

"I hereby give permission for the audio tape made for this research study to be also used for educational purposes."

Signature of Subject **Date**

IRB# 2003-04-0084

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The University of Texas at Austin

You are being asked to participate in a research study. This form provides you with information about the study. The Principal Investigator (the person in charge of this research) or his/her representative will also describe this study to you and answer all of your questions. Please read the information below and ask questions about anything you don't understand before deciding whether or not to take part. Your participation is entirely voluntary and you can refuse to participate without penalty or loss of benefits to which you are otherwise entitled. You will be given a copy of this form for your records.

Title of Research Study:

The Instructor's Role in the Development of a Classroom Community of Inquiry
(contained within IRB protocol of 2003-04-0084)

Principal Investigator, UT affiliation, and Telephone Number:

Stephanie Ryan Nichols, M.S., Doctoral Candidate, Department of Curriculum & Instruction, (512) 232-3957

(Dissertation Supervisor: Jennifer Christian Smith, Ph.D., Assistant Professor, Department of Curriculum & Instruction, (512) 232-9682)

Funding source:

None

What is the purpose of this study?

The purpose of this study is to investigate the teaching of an expert mathematics instructor, more specifically the instructor's role in this development of a classroom community of inquiry. Dr. Starbird has agreed to participate in this study.

What will be done if you take part in this research study?

You are being invited to participate in the interview portion of this study because you are enrolled in M328K in the Fall 2005 semester. Should you choose to participate, you will be interviewed approximately three times over the course of the semester. Each interview will take place on the UT campus at a time and a location to be agreed upon by you and the investigator. Each interview will last approximately one half hour. In these interviews, you may be asked to explain your personal thoughts about the number theory course you are taking.

What are the possible discomforts and risks?

Some people feel a certain degree of anxiety when being interviewed and recorded. We will make every effort to make you feel comfortable during the interviews. You are not required to answer every question asked of you.

If you wish to discuss the information above or any other risks you may experience, you may ask questions now or call the Principal Investigator listed on the front page of this form.

What are the possible benefits to you or to others?

There are no direct benefits to you for your participation. Future students may benefit from improved instructional techniques that may be developed from the results of this study.

If you choose to take part in this study, will it cost you anything?

No.

Will you receive compensation for your participation in this study?

You will receive $20 at the end of the semester to compensate you for approximately one and a half hours of time contributed to the study during the interviews.

What if you are injured because of the study?

There are no foreseeable physical risks associated with this study.

If you do not want to take part in this study, what other options are available to you?

Participation in this study is entirely voluntary. You are free to refuse to be in the study, and your refusal will not influence current or future relationships with The University of Texas at Austin.

How can you withdraw from this research study and who should I call if I have questions?

If you wish to stop your participation in this research study for any reason, you should contact: Stephanie Nichols at (512) 232-3957 or (512)773-7884 (cell). You are free to withdraw your consent and stop participation in this research study at any time without penalty or loss of benefits for which you may be entitled. Throughout the study, the researchers will notify you of new information that may become available and that might affect your decision to remain in the study.

In addition, if you have questions about your rights as a research participant, please contact Clarke A. Burnham, Ph.D., Chair, The University of Texas at Austin Institutional Review Board for the Protection of Human Subjects, 512/232-4383.

How will your privacy and the confidentiality of your research records be protected?

Authorized persons from The University of Texas at Austin and the Institutional Review Board have the legal right to review your research records and will protect the confidentiality of those records to the extent permitted by law. If the research project is sponsored then the sponsor also has the legal right to review your research records. Otherwise, your research records will not be released without your consent unless required by law or a court order.

If the results of this research are published or presented at scientific meetings, your identity will not be disclosed.

The interviews will be audio-taped and the cassettes will be coded so that no personally identifying information will be visible on them. You will be assigned a pseudonym, and you will be referred to using this pseudonym by the researchers when discussing the results of the interviews. The interviews will not be discussed with the instructors of the course until **after** final grades for the semester have been submitted. Cassettes of the interviews will be kept in a secure location and will only be heard by members of the research team. The cassettes will be transcribed and will be kept indefinitely for future reference and analysis.

Will the researchers benefit from your participation in this study?

No.

Signatures:

As a representative of this study, I have explained the purpose, the procedures, the benefits, and the risks that are involved in this research study:

Signature and printed name of person obtaining consent Date

You have been informed about this study's purpose, procedures, possible benefits and risks, and you have received a copy of this Form. You have been given the opportunity to ask questions before you sign, and you have been told that you can ask other questions at any time. You voluntarily agree to participate in this study. By signing this form, you are not waiving any of your legal rights.

Printed Name of Subject Date

Signature of Subject Date

Signature of Principal Investigator Date

Signature of Dissertation Supervisor Date

We may wish to present some of the tapes from this study at scientific conventions or as demonstrations in classrooms. Please sign below if you are willing to allow us to do so with the tape of your performance.

"I hereby give permission for the audio tape made for this research study to be also used for educational purposes."

Signature of Subject Date

IRB# 2003-04-0084

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The University of Texas at Austin

You are being asked to participate in a research study. This form provides you with information about the study. The Principal Investigator (the person in charge of this research) or his/her representative will also describe this study to you and answer all of your questions. Please read the information below and ask questions about anything you don't understand before deciding whether or not to take part. Your participation is entirely voluntary and you can refuse to participate without penalty or loss of benefits to which you are otherwise entitled. You will be given a copy of this form for your records.

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(Dissertation Supervisor: Jennifer Christian Smith, Ph.D., Assistant Professor, Department of Curriculum & Instruction, (512) 232-9682)

Funding source:

None

What is the purpose of this study?

The purpose of this study is to investigate the teaching of an expert mathematics instructor, more specifically the instructor's role in this development of a classroom community of inquiry. Dr. Starbird has agreed to participate in this study.

What will be done if you take part in this research study?

You are being invited to participate in the video portion of this study because you are enrolled in Dr. Starbird's section of M 328K in the Fall 2005 semester. We plan to videotape Dr. Starbird during the class meetings of this course for the entire semester. This means that your image and voice may be recorded for the purposes of studying the instructor's role in this class as well as the types of interactions that occur in this classroom setting.

What are the possible discomforts and risks?

Some people feel a certain degree of anxiety when they know that they are being recorded. We are interested in the way the instructor conducts class, not in the behavior of any particular individuals. We will make every effort to make sure the presence of the camera does not disrupt the class or make anyone uncomfortable. You are not required to appear on camera and can refuse to be recorded at any time, if you wish.

If you wish to discuss the information above or any other risks you may experience, you may ask questions now or call the Principal Investigator listed on the front page of this form.

What are the possible benefits to you or to others?

There are no direct benefits to you for your participation. Future students may benefit from improved instructional techniques that may be developed from the results of this study.

If you choose to take part in this study, will it cost you anything?

No.

Will you receive compensation for your participation in this study?

No.

What if you are injured because of the study?

There are no foreseeable physical risks associated with this study.

If you do not want to take part in this study, what other options are available to you?

Participation in this study is entirely voluntary. You are free to refuse to be in the study, and your refusal will not influence current or future relationships with The University of

Texas at Austin. If you do not wish to appear on camera, every effort will be made to avoid recording you, including pausing the recording of the camera when you are presenting a problem or otherwise the focus of attention. In the event that your voice is accidentally recorded, we will strike your comments from the transcript of that class session. In the event that your image is accidentally recorded, we will digitally blur your appearance in any tapes that will be viewed by someone not on the research team.

How can you withdraw from this research study and who should I call if I have questions?

If you wish to stop your participation in this research study for any reason, you should contact: Stephanie Nichols (512) 232-3957 or (512) 773-7884 (cell). You are free to withdraw your consent and stop participation in this research study at any time without penalty or loss of benefits for which you may be entitled. Throughout the study, the researchers will notify you of new information that may become available and that might affect your decision to remain in the study.

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If the results of this research are published or presented at scientific meetings, your identity will not be disclosed.

Your first name will be associated with your image on the videotapes, and these videotapes may be used in presentations at professional meetings or for educational purposes. Your last name will not be used. You will be assigned a pseudonym, and you will be referred to using this pseudonym by the researchers in any non-video-based presentations of the results and in publications. The videotapes will be viewed and transcribed by members of the research team. The videotapes will be kept in a secure location and will be kept indefinitely for future reference and analysis.

Will the researchers benefit from your participation in this study?

No.

Signatures:

As a representative of this study, I have explained the purpose, the procedures, the benefits, and the risks that are involved in this research study:

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Printed Name of Subject **Date**

Signature of Subject **Date**

Signature of Principal Investigator **Date**

Signature of Dissertation Supervisor **Date**

We may wish to present some of the tapes from this study at scientific conventions or as demonstrations in classrooms. Please sign below if you are willing to allow us to do so with the tape of your performance.

"I hereby give permission for the videotape made for this research study to be also used for educational purposes."

Signature of Subject

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Vita

Stephanie Ryan Nichols was born in Washington, D.C. on March 19, 1979, the daughter of Naomi Catherine Klaus and Thomas Glenn Nichols. After completing her work at H.B. Woodlawn Secondary Program, Arlington, Virginia, in 1997, she entered Colby College in Waterville, Maine. During the fall of 1999 she completed a semester abroad through New York University in Florence, Italy. She graduated from Colby College in May of 2001 Cum Laude with Honors and Distinction in Mathematics. The following year she was employed as a mathematics teacher at Grisham Middle School in Austin, Texas. In September 2002, she entered the Graduate School at the University of Texas at Austin. She received her Master of Science in Statistics in May of 2005 from the University of Texas at Austin. In the fall of 2007 she began teaching mathematics at Anoka-Ramsey Community College in Cambridge, Minnesota.

Permanent Address: 121 19th Ave SE, Cambridge, Minnesota, 55008

This dissertation was typed by the author.